MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 100 No. 7

MEETING OF 1940 MAY 10

Professor H. C. Plummer, President, in the Chair

The election by the Council of the following Fellows was duly confirmed :—

George Leslie Camm, B.A., University Observatory, Oxford, and Balliol College, Oxford ; and
Fred Hoyle, B.A., St. John's College, Cambridge.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Rev. William Kilshaw, St. John's Vicarage, Guernsey, C.I. (proposed by C. Anthony).

Fifty-one presents were announced as having been received since the last meeting.

The President drew attention to the urgent need for economy in the use of paper, and urged authors to compress as much as possible any manuscripts submitted to the Council for publication.

The Secretary announced that an application for lectures on astronomy to the troops in France had been received from the organisation "ENSA."

ACCRETION AND STELLAR ENERGY

R. d'E. Atkinson

Among the problems which, by common consent, would be solved or greatly eased if stars could sweep up matter at a very high rate, the question of the energy-supply for bright stars is certainly one of the most interesting and important. The difficulty presented by these stars, if practically all their energy is to be supplied by the transmutation of merely their original stock of hydrogen, without replenishment, has been generally recognised for some time, but is perhaps more acute than is often explicitly realised; in point of fact a star of absolute (bolometric) magnitude 0 has a maximum life, at its present rate of radiation, of about 3×10^9 years even if it started as pure hydrogen, and all brighter stars have less. This result refers to main-sequence stars, red giants being possibly rather more fortunate; but as the luminosity always goes up considerably faster than the mass it is clear that any really bright star, if it lives purely on its capital, will have an inconveniently short life.

Such progress as has been made with the theory of the source of stellar energy, since its inability to meet the needs of the bright stars was first stressed * to this extent, has not yet shown that transmutation is beyond question the principal factor even in the case of faint stars, though it seems likely here; but at least we can say that no other source has yet emerged to supplement it for the bright stars, unless central condensations can be so large as to make gravity important once more in them. This appears unlikely.†

To this extent then the problem has grown in urgency, and the question whether an important source of energy might after all be found in accretion deserves to be reopened. This will now be undertaken in a more general way than seems yet to have been attempted.

It appears in fact that we can make a useful survey of all possible formulæ for the rate of accretion, by any process whatever, within a certain very broad class. The most favourable of these formulæ can then be studied, from the point of view of what is the most to be hoped from it, quite apart from the question whether any known process would, in fact, result in such a formula. If it is found that the formula can predict a large enough supply of hydrogen, while no others can, this would certainly be grounds for supposing that some process leading to it actually is operative; if, on the other hand, no formula within the entire class considered could predict a large enough accretion to be of any use, the emphasis would be shifted to processes (if any) lying outside this class.

* R. d'E. Atkinson, *Ap. J.*, **84**, 83, 1936.

† J. Wasiutynski, *M.N.*, **100**, 362, 1940.

A very powerful weapon, whenever it is applicable, for the kind of purpose in view is, as is well known, the dimensional analysis of elementary physics. If, on general grounds, a quantity A can be expected to be given by a product-formula of the type

$$A = k \,.\, a^x b^y c^z \,.\,.\,. \qquad (1)$$

involving parameters a, b, c, . . . and a dimensionless constant of proportionality k, then if there are only three such parameters, and if the determinant of the solution does not vanish, the corresponding exponents x, y, z can be rigorously determined. If there are more than three variables involved, the problem is indeterminate, but if there are only four there is a one-parameter array of solutions, and it is often possible to draw important conclusions. In either case the practical value is due to the further consideration that, if the exponents are determinable, we then have also the actual order of magnitude of A itself. The reason for this last statement, which I believe is due to Einstein, is that hardly any (classical) argument is known which introduces *large* pure numbers at any point, and of the various small numbers, such as π, 2, etc., that easily may be involved it is unlikely that very many more will come in the numerator than the denominator, or *vice versa*. Thus k will be of the order of magnitude 1 or $10^{\pm 1}$ roughly. In actual fact very few formulæ do involve a larger pure number than $4\pi^2 \cong 10^{1\cdot 6}$ (especially if $\hbar$ is used instead of $h/2\pi$).

In the case of accretion by a star there definitely are more than three factors that may well be involved, and of course it has not been shown that a product-formula of the type (1) is unquestionably applicable at all. We shall, however, assume that it is, and study the consequences.

The factors which it is reasonable to suppose may have to be considered are M, the mass of the star ; R, its radius ; v, its speed through the diffuse matter ; ρ, the density of the diffuse matter ; and γ, the constant of gravitation. It is possible that such quantities as the temperature or molecular weight of the diffuse matter ought also to be considered ; if so, the present attack breaks down. However, formulæ in which the accretion appears as proportional to a power of these quantities seem not to have been suggested as yet, and we shall assume they are not concerned except perhaps as small corrections. The temperature of the star is also unlikely to be important ; or, if it is, we are again defeated.

Even so, five factors constitute a complex problem in general. However, it seems fairly safe to assert that ρ must be involved to the power $+1$ only. The square of a gas-density may appear, for example, where the total numbers of intermolecular collisions in the gas are a deciding factor, as in recombination problems ; but in the case of accretion it seems unlikely that anything except a direct proportionality with ρ would appear in the final formula. (See also below.) We shall therefore assume that the rate at which a star gains mass is given by

$$A \equiv \frac{dM}{dt} = k\gamma^w R^x v^y M^z \rho \qquad (2)$$

with $|\log k| < 1\cdot 6$.

Since the dimensions of γ are $M^{-1}L^3T^{-2}$ we have at once $w=z$. Accordingly $L^{x+y+3z-3}=T^{y+2z-1}$, and the possibilities are represented by the straight line obtained from any two of the three interdependent equations

$$\left.\begin{aligned} y+2z&=1,\\ z+\ x&=2,\\ 2x-\ y&=3. \end{aligned}\right\} \tag{3}$$

In other words, we may rewrite (2) in the form

$$A=k(\gamma M)^z R^{2-z}v^{1-2z}\rho. \tag{4}$$

This represents all possible variations of (2) that are dimensionally permissible, and (2) itself represents all variations of (1) that seem physically reasonable unless such factors as the constitution or state of the diffuse matter are to appear explicitly in the accretion formula.

The actual values of M, R, v and ρ that it is most advantageous to insert in (4) will of course depend on z; but there is not in fact a very wide range available, and it is helpful to insert provisional standard values even before discussing z. This procedure is unorthodox, but in the present instance it is convenient and harmless. The effect of changing the values from the standard ones, to such extent as is actually permissible, is easily seen in any particular case.

For M we may usefully take a mass larger than the Sun's (since it is for heavy stars that the energy problem is serious), but still not a very high figure, as very massive stars are extremely rare. We shall take $M=10^{34}$ gr. for a first survey. Large values of the accretion will be favoured by taking R as large as possible, and v, in most cases, as small as possible; we shall set $R=10^{13\cdot5}$ cm. (red supergiant) and $v=10^{5\cdot6}$ (4 km./sec.). ρ varies in different parts of the galaxy, but as it enters linearly we shall be concerned with its mean value over long paths; it is barely possible that regions of unexpectedly high density might regularly have to be included, and we shall take $\rho=10^{-22}$ even though this probably errs appreciably on the side of generosity (especially as only the hydrogen-density is of value for our present problem). We have accordingly *

$$A=k\ .\ \text{ant}\,[26\cdot8z+13\cdot5(2-z)+5\cdot6(1-2z)-22]=k\ .\ \text{ant}\,[2\cdot1z+10\cdot6]. \tag{5}$$

This is simpler than (4) for arithmetical estimates; the logical possibilities are, however, better shown in equations (3) and (4), and will be discussed with their help.

(*a*) Let us first suppose that the physical target-area presented by the star is a determining factor, so that A is proportional to R^2. From (3) or (4) we see then that $z=0$, so that, within the range of possibilities covered by (2), if the accretion is proportional to the target-area it is necessarily independent of the stellar mass. (This is more or less evident physically also, but the dimensional argument is rigorous.) We have in fact

$$A=kR^2v\rho$$

* Ant [x] (antilogarithm of x) is written here for 10^x, just as exp [x] frequently is for e^x.

and, in the only obvious actual case, $k=\pi$. Inserting $k=\pi$ and $z=0$ in (5) we have $A=10^{11\cdot1}$ gr./sec., which will be multiplied by a factor of 5 if we assume v is 20 km./sec., but cannot otherwise be appreciably increased; if R is put equal to the radius of an A- or B-type star the figure is in fact very considerably smaller.

If accretion is to be of practical interest from the point of view of stellar energy (or indeed almost from any point of view) considerably larger values than 10^{11} will be required; for practically any purpose it is in fact necessary to add to a star, during its life, a mass comparable with its existing mass. The problem of energy-generation being most acute for the massive stars, we may thus, for the present, demand of a "useful" accretion process that it shall at any rate have some prospect of adding 10^{34} gr. in $10^{9\cdot5}$ years, or $A=10^{17}$ gr./sec. as an average over this time. (We shall see below that even more is in fact needed.) Thus no processes, of any sort, in which the matter swept up is proportional to the target-area of the star are of appreciable interest whatever the proportionality-constant (and whatever the mass of the star). This is in fact a result of some generality, for from the point of view of dimensional analysis R need not be the radius of the *star*; it could be the radius of any relevant appendage, such as a planetary system or a zodiacal light; the accretion-process itself may still be quite undefined, and we can nevertheless say that if the assumptions on which (2) is based hold, no accretion process, whose probability is proportional to the square of *any* length associated with the star, can have much effect unless the length in question is something like four hundred times as large as we have supposed; this would nearly be comparable with the radius of a planetary *nebula*.

(*b*) The suppositions that A is proportional to M, to R, or to $1/v$ are evidently equivalent; they each lead to

$$A=k\gamma MR\rho/v,$$

which is the most usually accepted relation *, if one inserts the appropriate value of k (i.e. $k=2\pi$). It is well known that the resulting rate of accretion is still too small to be of appreciable assistance. Putting $z=1$ and $k=2\pi$ in (5) we have $A=10^{13\cdot5}$, and there is clearly no way to increase this value very much.

(*c*) It would be most helpful if accretion could be very appreciably more important for moderately massive stars than for light ones. Larger values of z evidently also increase A, and we will therefore try next a proportionality with M^2. From (3) we obtain at once $x=0$, so that the stellar radius is now without influence; in fact

$$A=k\gamma^2M^2\rho/v^3. \qquad (6)$$

A formula of this type has recently been proposed by Hoyle and Lyttleton †, who have also sought to explain a very wide variety of phenomena with its help. The arguments from which they derive it appear open to very serious

* See, for example, Eddington, *I.C.S.*, p. 391.

† F. Hoyle and R. A. Lyttleton, *Proc. Camb. Phil. Soc.*, 35, 405 and 592, 1939.

question in a number of ways *, but some formula of this class might of course prove to be correct none the less. Inserting $z=2$ in (5) we now obtain $A=k\,.\,10^{14\cdot8}$, with a rather marked dependence on v. Allowing for the fact that the value of k may exceed 10, and for the possibility that, at least occasionally, v might be even less than 4 km./sec., it is evident that processes leading to a rate of accretion independent of the stellar radius (or proportional to the square of the mass) may at least intermittently supply fairly large amounts of matter. Whether, even so, the amount can be sufficient to explain the stellar energy requires further study; certainly it can at most be sufficient only if the star's speed is very small.

(*d*) A more pronounced dependence on M than is given by this third class of formulæ seems unlikely, for it would involve variation with a negative power of R (Eq. (4)). Now, for the large rates of accretion desired, it is essential that the star should be able to call on matter at very great distances from its path through the gas; it is difficult to envisage any calling-mechanism that would depend on the star's radius at all, or any method of selecting the atoms called, once they have been drawn in to close quarters, that would be *less* effective for large stars than for small. Thus it seems improbable that x can be negative.

We see then that the possibilities are in fact surprisingly restricted. Log k is unlikely to be greater than 1·6 (Hoyle and Lyttleton's value is $k=4\pi$); $\frac{dx}{dz}$ and $\frac{dy}{dz}$ are both negative; and we have in fact adequately sketch-mapped the entire region between $x=0$ and $z=0$. Negative values of x being unlikely, and negative values of z being not only unlikely but also useless if we could get them (since, from (5), they would obviously lead to very small numerical values, and in any case the massive stars are the ones we are interested in), we are effectively confined, in the case of large accretion, to formulæ characterised by values of z near $+2$. As non-integral powers of the stellar mass also appear definitely improbable in a case of this sort, formulæ of the type

$$A=4\pi^2\gamma^2M^2\rho/v^3 \qquad (7)$$

are the only ones we can expect, if the case is covered by (2) at all, and *if accretion really is large.* We will therefore study in detail to what extent accretion according to this formula would solve the problem; we have definitely set $k=4\pi^2$, since it is unlikely to be more.

The factor v^{-3} does, of course, mean that when a star is moving really slowly through the diffuse matter, large rates of accretion are possible; if it could move for the greater part of its life at a speed of 2 km./sec., it could gather a little more matter (if ρ really were as large as 10^{-22}) than we have stipulated. Such speeds are unusual, and we shall confine ourselves to values for which we have definite experimental evidence; nevertheless, the examination of some specific cases is instructive even though the prospect is not hopeful.

* R. d'E. Atkinson, *Proc. Camb. Phil. Soc.*, in print. The essential physical objections involved seem not to be contested by the authors (*Ib.*, in print).

We will consider first the case of a B star of mass $10\odot$, to which corresponds an absolute magnitude of about $-3{\cdot}7$.* Such stars exist in quite considerable numbers. The total life of the star in the absence of accretion, if transmutation of hydrogen is its sole source of energy, is ant $[34{\cdot}30 - 2{\cdot}16 + 20{\cdot}95 - 33{\cdot}58 - 0{\cdot}4 \times 8{\cdot}55] = 10^{16{\cdot}09}$ sec. $= 10^{8{\cdot}6}$ years (putting the Sun's absolute magnitude at 4·85). Thus, even neglecting all their radiation in early life, these stars must all have attained their *present masses* not more than $10^{8{\cdot}6}$ years ago; if they can be regarded as still building themselves up, they may have been able to radiate, as smaller stars, at earlier times, but under no circumstances can they have performed a large fraction of their accretion earlier. If their change of mass has not in fact been large in this period, their total life cannot even extend backwards much beyond its commencement at all. Their present velocities should therefore be a good guide to the value of v to adopt for most of their accretion-life; moreover, the mean value of ρ during that time must also be roughly the mean value for the regions where they now are, and this is probably well below 10^{-22}. However, we shall pass over this latter point for the moment.

$10^{8{\cdot}6}$ years is definitely short compared with the probable age of the galaxy; it is conceivable that these stars have recently been able to increase their mass considerably, and that they are now living mainly on their capital like other stars, but it would be more satisfactory in such a case as this to suppose, if possible, that their present rate of accretion is about enough for their present rate of radiation, so that they are approximately living on their income. The desirability of this is increased by the consideration that if we can account for stars of this luminosity in this way we can also account for all brighter ones, as well as all fainter ones; for the $M - L$ relation approximates to $L \propto M^2$ at large masses, so that rate of accretion and rate of radiation could go up together. However, it seems that we must forego this hope.

The rate of accretion of mass must be $1/{\cdot}007 = 10^{2{\cdot}16}$ times the rate of loss of mass by radiation, if all the energy comes from the transmutation of captured hydrogen. We require then

$$A = \text{ant}\,[33{\cdot}58 + 0{\cdot}4 \times 8{\cdot}55 - 20{\cdot}95 + 2{\cdot}16] = 10^{18{\cdot}2}\ \text{gr./sec.},$$

assuming that the whole of the mass swept up is hydrogen. If in fact there is 1 atom as heavy as calcium to each 40 of hydrogen (as there well may be), only half the mass is hydrogen and a total of $10^{18{\cdot}5}$ gr./sec. will be needed. It does not seem possible to stretch (7) to cover these figures. If we retain the figure $v = 10^{5{\cdot}6}$ we have

$$\rho = \frac{v^3 A}{4\pi^2\gamma^2 M^2} = \text{ant}\,[16{\cdot}8 + 18{\cdot}2 - 1{\cdot}6 - 1{\cdot}6 + 16 - 2 \times 34{\cdot}3] = 10^{-20{\cdot}8}$$

for the density of hydrogen alone; if, on the other hand, we set $\rho = 10^{-22}$ we have $v = 10^{5{\cdot}2} = 1{\cdot}6$ km./sec. The order of magnitude, in the second case, is satisfactory, but the actual value cannot really be reconciled with radial

* This is taken from an empirical curve drawn to fit the points plotted in Eddington, *I.C.S.*, p. 153, together with the correction-term for surface-temperature.

velocity data. If (as is probable) ρ is really less than 10^{-22}, this difficulty is of course increased.

We are thus forced to try the less attractive hypothesis that these stars are living more on capital than on income, but have only recently become really spendthrift. Even by this means we cannot gain a large factor; for if the radiation approaches 145 times the energy available per second from *captured* hydrogen the star will never actually gain mass, and must therefore always have been as spendthrift as it now is. There is one possible loophole here; if the effect of changing molecular weight on the $M-L$ relation were considerable, the star might in its early life have radiated less even with the same mass, and could therefore also have gained some mass for a while. However, observation suggests that the $M-L$ relation is not very sensitive to molecular weight for stars as they now are.

The question is, essentially, whether a sufficiently rapid rise of mass is possible in any case. We shall attack this by inquiring how successfully a star of absolute magnitude about -1 could increase its mass and luminosity to the figures we have been discussing. Such stars have under present circumstances masses that do fit Eddington's relation, and it is rather difficult to suppose that they were all different in the past. Assuming a surface-temperature about 16,000°, the mass will be $M=10^{33\cdot3+0\cdot55}$ approximately.* The total energy available, if all this is hydrogen, is ant $[33\cdot85-2\cdot16+20\cdot95]$ $=10^{52\cdot64}$ ergs. The rate of radiation is ant $[33\cdot58+0\cdot4\times5\cdot85]=10^{35\cdot92}$ ergs/sec., and the life, in the absence of accretion, would thus be ant $[52\cdot64-35\cdot92-7\cdot5]=10^{9\cdot2}$ years. Thus a star of this type needs some accretion if it is to live $10^{9\cdot5}$ years, and in fact, since twice the hydrogen would be needed even at constant radiation-rate, while twice the total mass will in its turn result in a final radiation-rate four times as large as the initial one, considerably more than doubling the mass will actually be necessary.

The rate of gain of mass is proportional to M^2; the rate of loss by radiation will also vary approximately as M^2, though the effects of changing surface-temperature and changing molecular weight are somewhat uncertain, especially when it is remembered that hydrogen is being supplied at the surface of the star and used up mainly near the centre. We shall assume, however, that a proportionality with M^2 is roughly correct. Then

$$\frac{dM}{dt}=\left(\frac{4\pi^2\gamma^2\rho}{v^3}-\frac{10^{35\cdot92}}{(10^{33\cdot85}c)^2}\right)M^2$$

$$=\{\text{ant}\,[1\cdot60+1\cdot64-16-22-16\cdot80]-\text{ant}\,[35\cdot92-20\cdot95-67\cdot70]\}M^2$$

$$=(10^{-51\cdot56}-10^{-52\cdot7})M^2=10^{-51\cdot60}M^2.$$

Thus such a star will gain *mass* nearly as fast as if it were not radiating any mass at all, but still not nearly fast enough for the *hydrogen* gained to supply the *energy* radiated; $A-L/c^2$ is positive, but $A-145L/c^2$ is negative. The actual time needed to double the mass is given by

$$\frac{dM}{M^2}=10^{-51\cdot60}dt,$$

* Obtained as before.

so that

$$\frac{1}{M_0} - \frac{1}{M} = \frac{1}{2M_0} = 10^{-34\cdot15} = 10^{-51\cdot60}t,$$

$$t = 10^{17\cdot45} \text{ sec.} = 10^{9\cdot95} \text{ years.}$$

It is evident that even at the initial rate of radiation the hydrogen would all be used up long before this point could be reached, while in fact the rate of radiation should be increasing as well ; thus the star could not remain bright without calling on some other source of energy.

In point of fact, if this particular case is worked out rigorously, a rather tedious calculation shows that the life when accretion is allowed for is $10^{9\cdot22}$ years, *i.e.* just the same as before. The increase of hydrogen is almost exactly balanced by the increased radiation that accompanies the larger mass. In general this balance would not be exact, but it does appear in all cases that accretion is of very little practical help unless it really can supply almost the entire needs of the star for its whole life ; this we have already seen appears impossible.

In their study of possible dynamical effects of accretion Hoyle and Lyttleton appear to have ignored the loss of mass due to radiation. They find that on this basis dynamical effects of important size could be produced if the total life of the universe could be about $10^{10\cdot5}$ years, and they express the opinion that it really has already existed as long as this, "or even 10^{11} years." The present inquiry shows that while it may, with the values so far assumed, be permissible dynamically to ignore the loss by radiation, the question of the source of stellar energy would be very seriously aggravated by the assumption of so long an existence for stars that are now bright. However, it should also be emphasised that the data have been somewhat strained as it is, and it is quite easy to make out a case against the view that stars could even gain any mass at all, radiation being allowed for. We might, for example, put $\rho = 10^{-23\cdot14}$; or again $\rho = 10^{-22\cdot70}$, which is in fact Hoyle and Lyttleton's figure, and $v = 10^{5\cdot75}$ ($= 5\cdot6$ km./sec.), which is certainly not excessive even for bright B stars ; in either case then the two terms in the above equation for dM/dt are equal, the star will never gain mass, its total life will not exceed that in the absence of accretion by as much as 1 per cent. (if hydrogen is the energy-source), and there will be no dynamical effects whatever the time-scale and whatever the energy-source. In other words, the rate of accretion given by the Hoyle-Lyttleton formula (and any others of the type (6)) can never (except for unusually slowly moving stars) be so large that the correctness or incorrectness of the formula can be tested by its dynamical effects at all.

The present purpose, however, has not been a dynamical study ; the inquiry has been directed to the question whether the energy requirements could be met by the accretion of hydrogen. It appears that they could not. Incidentally there is then little to support either the assumption that the matter swept up would in fact be hydrogen, or the assumption that accretion is really large.

The question whether (2) can be in any way extended seems to be physical

rather than algebraical. Formally, however, we may try the effect of putting $A \propto \rho^2$ and $A \propto \rho^0$. In the first case we obtain

$$A = k\gamma^{z+1}R^{4-z}v^{-2z-1}M^z\rho^2 = k \text{ ant } [2{\cdot}1z - 2{\cdot}8] \tag{8}$$

if we employ the same numerical values as before; it is quite plain that unless z is improbably large, A is now bound to remain very small. In the second case ($A \propto \rho^0$) we obtain

$$A = k\gamma^{z-1}R^{-z}v^{-2z+3}M^z = k \text{ ant } [2{\cdot}1z + 24{\cdot}0]. \tag{9}$$

If neither the exponent of R nor the exponent of M is to be negative, this reduces to $A = kv^3/\gamma$, and, while the numerical value ($10^{24}k$) is now more than adequate, the formula itself, in spite of being dimensionally permissible, is obviously absurd.

It would seem nevertheless that there might be some possibility of a formula in which A varied as some power of ρ between 0 and 1, and which would give a sufficiently large numerical value; thus we are forced to consider an objection which applies to all cases in which the exponent of ρ is not $+1$ exactly.

It is clear that we can always regard the star as sweeping up all the diffuse matter inside a cylinder of radius S say, and none outside it, so that S is defined, whatever the actual accretion process, by the equation

$$A = \pi S^2 \rho v.$$

S is then the "effective capture radius"; it may or may not be somewhat fictitious actually, depending on whether some of the matter originally nearer the star's path than S fails in fact to get captured, while some that was farther away succeeds. It is at once evident that, if A is (physically) proportional to any power of ρ except $+1$, S is itself dependent on ρ. This statement is quite general, even if the accretion process involves quantities not included in (2), such as the molecular weight of the diffuse matter, or indeed even if A is not given (physically) by a product-formula of the type (1) at all. Now it appears almost certain, on general grounds, that if the effective capture-radius S is in fact dependent on the density of the matter to be captured, the matter must itself be interfering with (or helping) its own capture; if in particular $A \propto \rho^p$ where $p < 1$, then S gets smaller as ρ increases, so that for the same values of M, R, v, and any other relevant parameters, the star's effective "reach" is less at high densities than at low, *i.e.* at high densities some of the matter within its reach is not grasped. This is evidently conceivable, but it is very difficult to suppose that it could occur unless, for example, the pressure, or the molecular diameter, or perhaps the opacity, of the gas is relevant in addition to M, R, and v. If such quantities are in fact relevant, the present type of analysis leads to equations that are very considerably under-determined, and in particular neither formulæ (5), (8) or (9), nor any interpolation between them, would hold any longer.

Thus we can make certain statements with a high degree of confidence, but the restrictions on their validity are also quite definite and fairly extensive. Provided that the accretion does not depend formally on any parameters

beyond M, R, v, ρ and γ, and provided that it is governed by a formula of the type (1) at all (not *necessarily* (2) even, *a priori*), we can say that no formula is possible which will give a high enough rate of accretion to account for the energy of the bright stars, at least with any values of these parameters that seem permissible at present. Further, the rate of accretion cannot even be high enough to produce the dynamical effects considered by Hoyle and Lyttleton, except for very slow-moving stars, and even then only at the price of extending the time-scale to 10^{10} years or more; this involves reducing considerably further the upper limit to the luminosity that can be accounted for by the transmutation of hydrogen, even with the help of accretion. It is perhaps arguable that since the brightest stars seem in any case impossible to explain in this way there can be little harm in thus extending the difficulty; but it is equally true that since even the dynamical properties of stars of normal and larger velocity can certainly not be explained there is little advantage in explaining the slow ones. It must, however, be added that, except in the case of exceptionally high interstellar density coupled with exceptionally low stellar velocity, accretion (subject to the same limitations as before) cannot in any case even supply as much mass as the star loses by radiation; no dynamical effects are then to be expected at all, except those of the same sign as would occur without accretion, however long the time-scale. Observationally it is highly improbable that stars moving at 2 km./sec. should be able to increase their masses rapidly, while stars moving at 6 km./sec. should be totally unable to increase them at all; there is no discernible barrier here, as there would be if accretion obeyed any formula of the type (2) and were also really large.

Summary

The methods of elementary dimensional analysis are applied to the question of determining the maximum possible rate of accretion of diffuse matter by stars. It is found that if the rate of accretion is given by any product-formula involving only powers of the star's mass and radius, its velocity with respect to the diffuse matter, the mean density of the matter, and the constant of gravitation, then the problem of the radiation of the bright stars cannot be solved by any admissible accretion formula at all; indeed, considerable dynamical results are possible only if extreme values of the parameters are postulated, and even then only if the time-scale is extended to 10^{10} years or more, in which case the radiation problem is further aggravated. Thus either some such additional factor as the molecular nature or molecular state of the diffuse gas is explicitly relevant, or else the rate of accretion is not given by any simple product-formula at all, or finally the total radiation of the bright stars requires some additional source of energy beyond that provided by all the hydrogen they initially possessed and all they can subsequently collect.

Royal Observatory, Greenwich:
1940 *April* 4.

SEASONAL VARIATION IN THE HEIGHT OF METEORS

R. A. McIntosh

Introductory.—The seasonal variation in the heights of meteors has already been exhaustively examined by E. Öpik *, and an explanation is necessary in producing additional data on the subject.

Acting upon a suggestion early in 1937 by Dr. D. F. Martyn, the noted radio-physicist at Sydney University, that he would be interested to learn whether meteor heights revealed any seasonal variation in the atmosphere, and unaware that the Harvard Observatory Meteor Expedition had included a similar research in their programme, I began the collection of the necessary data and the formation of a catalogue of real paths. While this was proceeding Öpik published his interesting and valuable paper.

Reluctant to abandon a research which had occupied a considerable portion of my time, I finally decided to continue my task in the hope that it might provide supplementary confirmation of Öpik's conclusions. My decision was a fortunate one in that, not only have I been able to confirm the seasonal variation discovered by Harvard in the centre of the observed paths, but also have been able to demonstrate this variation at the beginning- and end-points as well, making the reality of the phenomenon more certain. A number of other interesting features not visible in Öpik's data can also be demonstrated.

The present paper treats in detail only three phases of the matter published by Öpik, namely, the seasonal variation in height, the abnormality of shower meteors, and the method of handling the data. Single-handed, I have been compelled to neglect the formidable task of introducing all the corrections used by Öpik, such as allowance for magnitude, elongation from apex, etc. While frequent comparisons between Öpik's conclusions and my own will be necessary throughout this paper, it must be borne in mind that there are considerable differences in treatment, material and, in some cases, in definitions used.

Grouping of Available Data.—The material used in the present discussion, all the published meteor heights it was possible to discover and obtain, was as follows :—

W. F. Denning : *Monthly Notices*, **57**, 3, 164 ; **72**, 5, 423 ; **76**, 3, 219 ; *Journ. B.A.A.*, **27**, 252 ; **28**, 118 ; **29**, 20 ; **32**, 175 ; **32**, 254. Total, 950 meteors.

British Astronomical Association : *14th* and *15th Memoirs of Meteor Section* ; Interim Reports in vols. **30–32** and **47–48**. Total, 310 meteors.

A. King : *Meteor Notes* in *Observatory*, vols. **25** and **41–53**. Total, 199 meteors.

* *Tercentenary Papers, Harvard Observatory*, **105**, 549, 1937.

A. S. Herschel : *British Ass. Reports*, 1864–80. Total, 285 meteors.
American Meteor Society : *Reprints*. 542 meteors.
C. C. Wylie : *Contributions Iowa Obs.*, Nos. 1–7. 18 meteors.
Hoffmeister-von Niessl Catalogue. Total, 370 meteors (duplications being removed).
Tashkent Observatory Publications, IV, No. 2, and V, No. 3. Total, 47 meteors.
Japan : *O.A.A. Circ.*, 2 ; *Memoirs Jap. Ast. Soc.*, **3**, 99 ; **3**, 339 ; **4**, 136 ; **5**, 115 ; *Kwasan Bull.*, Nos. 298, 311, 312, 313, 324. Total, 130 meteors.
Miscellaneous foreign sources. Total, 23 meteors.
Grand total, 2856 meteors.

Throughout the present paper the data have been subdivided into three groups, as follows :—

Group I.—Denning's published data. This material, being the largest quantity provided by one computer, can be used apart from the remainder if the greatest homogeneity is desired.

Group II.—Denning's data modified by the addition of real paths by other British computers, the B.A.A., Herschel, and King. Being performed on similar lines to Denning's, the two may be combined if larger totals are desired at the loss of homogeneity.

Group III.—The whole of the available data, both British and foreign.

Three points were used in the path of each meteor, the height at appearance (*Ha*), the height at mid-point (*Hm*), and the height at disappearance (*Hd*). The second point (*Hm*) was added to facilitate comparison with Harvard's results, for their investigation revealed a seasonal variation only in the height of the centre of the trail (*Hc*).

As by far the greater proportion of the data was published in miles, it was more convenient to do all the computations in that measure, rather than in the more favoured metric system. The few heights which were originally given in kilometres have been dealt with as such, and the averages transformed into miles at a later stage.

Data Rejected.—Any height recorded as greater than 125 miles (or 200 km.) was excluded from the material, together with the middle height based upon it. Meteor computers view with grave suspicion any point of apparition which comes out at over 100 miles, and generally append a special note concerning the accuracy of the observation. By setting the limit at 125 miles, borderline cases remain in, but physically impossible heights are ruled out. Paths for which two values were published were also excluded, as also were cases in which the computer indicated his doubts in the accuracy of the figures. Naturally, as the investigation concerned seasonal variations, any meteors observed in the southern hemisphere were also excluded. No path was excluded, however, because the end-point was higher than the beginning-point. It would be impossible to

say which point was erroneous, and the difference between the two was never great. Possibly in some cases the rise in height was real.

Method of Treatment.—Öpik's use of the harmonic mean in determining mean heights from the data obtained by the Harvard Meteor Expedition must have come as a surprise to many astronomers engaged in meteoric researches. Harmonic means had been used in discussions of meteor velocities by Fisher and Maltzev, but the arithmetic mean had been so commonly employed in determining heights that its use was regarded as firmly established.

Admittedly the question of the best procedure in such analyses is a difficult mathematical problem, which in this instance is complicated by special physical features. In avoiding one error, however, we must not fall into another. Because Öpik used the harmonic mean in the analysis of his data, it does not necessarily follow that this treatment should be followed as blindly in future discussions of meteor heights as the arithmetic mean has been in the past.

From a statistical point of view, the choice of the arithmetic, harmonic, or any other mean depends upon the nature of the frequency curve of the data employed. Table XXXIV on p. 595 of Öpik's paper reveals that in the Harvard data the frequency distribution of heights for the four groups considered revealed a pronounced skewness about the mode, the skewness being positive and toward the greater heights. In such a case the use of the harmonic mean, giving less weight to the higher points than would the arithmetic mean, would appear to have some justification.

The Harvard data, however, consisted of essentially normal meteors gathered in routine observations from day to day over a long period, and every meteor doubly observed provided a unit in the data discussed.

A different state of affairs is revealed in the collection of the data used in this paper. While the British material contained a proportion of normal meteors obtained by dual observation by experienced meteor observers, a number of cases were those of fireballs, detonating meteors or meteorites, whose brilliancy and other abnormalities made them suitable objects for the collection of observations from the general public, while the Hoffmeister-von Niessl catalogue, as its title implies, consisted solely of great fireballs. Such objects were definitely not normal in that, either from their composition, velocity or other reasons, their luminous paths were unduly prolonged and they penetrated to much lower levels than the average meteor.

The harmonic mean gives such great weight to a low point as to disturb seriously the true trend of a seasonal curve based on data such as those available to me. Take, for example, a number of normal meteors observed by the British Astronomical Association and A. King in one month (February). These 11 meteors had end-points ranging from 25 miles to 55 miles, but there was one low end-point of 9 miles. If that one low point is excluded the arithmetic mean is 39·9 miles and the harmonic mean 36·0 miles, a difference of 3·9 miles. The inclusion of the abnormal meteor, however, makes the arithmetic mean 37·1 miles and the harmonic mean 28·6 miles, a difference of 8·5 miles.

Öpik uses the difference between arithmetic and harmonic mean in such cases as an index to the observational error. Therefore, on his reasoning, the index of error in the above set of real paths was 3·9 miles (or 10 per cent. of the mean height) with the abnormal meteor excluded, which becomes 8·5 miles (or 25 per cent. of the mean height) when that particular meteor is included. Yet the meteor in question was not responsible for more than a 200 per cent. increase in the error of observation. It was probably as well determined as any other meteor in the group and owed its low end-point not to any error in observation or computation but because it actually penetrated deeper into the atmosphere.

From the physical point of view it is obvious to the experienced meteor worker that the harmonic means for those months which contain a few low end-points will be unduly depressed in relation to those of neighbouring months in which, by chance, all the meteors considered are normal. The symmetry of a curve would be badly upset by the use of the harmonic mean in such circumstances.

It becomes clear, therefore, that in any attempt to present a true picture of a given set of material an examination should be made of the frequency curve of the data employed. For my purpose I selected all the real paths published by Denning as being a fair sample of the whole. These curves for point of appearance and disappearance are shown in fig. 1 (p. 514) and are found to be essentially symmetrical about the mode. In such circumstances the arithmetic mean is the one which gives the best result, and the use of the harmonic mean would be less satisfactory.

A purely physical question arises in the examination of the frequency curve for *Hd*. Here the normal trend is distorted by the discontinuity first described by Lindemann and Dobson *, unduly depressing the curve in the region about 37 miles and unduly raising it below that height. Probably if the atmospheric discontinuity were absent the curve would be symmetrical like that for *Ha*, but, being negatively skewed, it is in any case more suited to the use of the arithmetic than the harmonic mean.

Average Heights of Meteors.—Before proceeding further I give the average heights of the meteors used in this work, which will be useful later for comparison with the seasonal heights.

TABLE I

Mean Heights of Meteors (Miles)

	Ha		*Hm*		*Hd*	
	No.	*H*	No.	*H*	No.	*H*
		m		m		m
Showers	624	76·0	620	64·0	638	53·9
Non-shower :						
Group I	820	67·7	821	56·0	830	44·9
,, II	1427	68·3	1432	56·3	1458	45·6
,, III	1996	68·0	1980	56·2	2113	44·1
All meteors	2620	70·0	2600	58·0	2851	44·7

* *Proc. Roy. Soc.*, A, **111**, 411, 1923.

Effect of Shower Meteors.—When the average values of meteor heights are plotted for inspection (fig. 2) it is immediately noticed that in the months of April, July, August, October and November marked peaks disturb the general trends of the curves, and that these peaks are visible in all three curves, *Ha*, *Hm* and *Hd*. The fact that the principal annual showers, the Lyrids, Perseids, Orionids and Leonids, occur in those months indicated

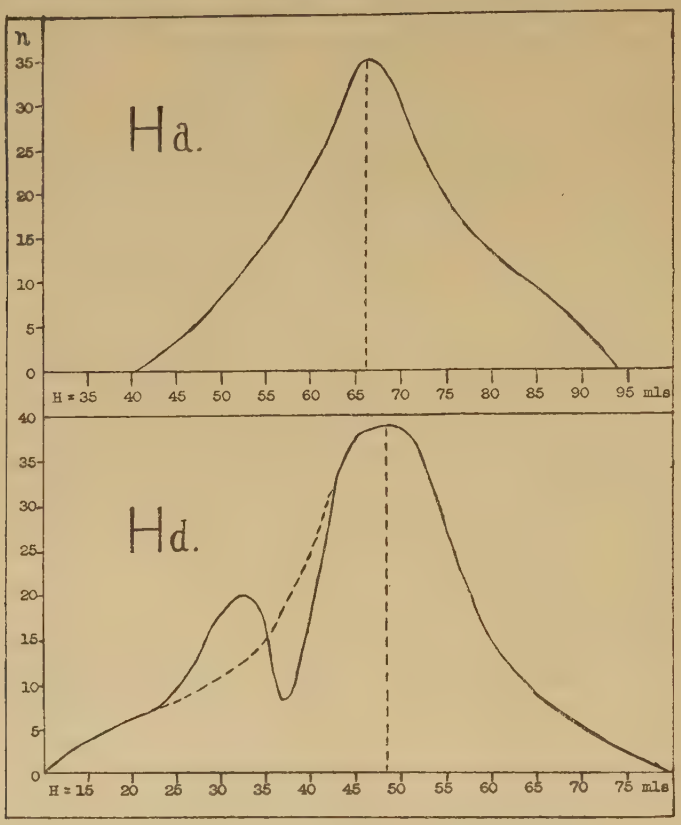


Fig. 1.—*Frequency Curves of Denning's Heights.*

that for some reason not immediately obvious the shower meteors differed from the remaining meteors and required to be excluded from the data.

It was rather difficult to decide which showers to exclude. Öpik defined a shower meteor as one emanating from or passing near a "group radiant," but the English practice of publishing the radiant point of any real path, irrespective of whether it was a known radiant or not, would have meant a personal investigation of nearly 3000 cases. That achieved, the thorny question of which of these radiants were real and which fictitious would have remained to be solved.

Obviously the Lyrids, Eta Aquarids, Perseids and Leonids would have to be excluded from the discussion, as they are known to be connected with periodic comets and therefore travelling with elliptical velocity, in contradistinction to the hyperbolic velocities of sporadic meteors found theoretically

by Hoffmeister * and observationally by Harvard.† There would still remain, however, the Quadrantid, Delta Aquarid and Orionid showers, which annually give displays comparable to the former showers, but whose identity with periodic comets has not been established.

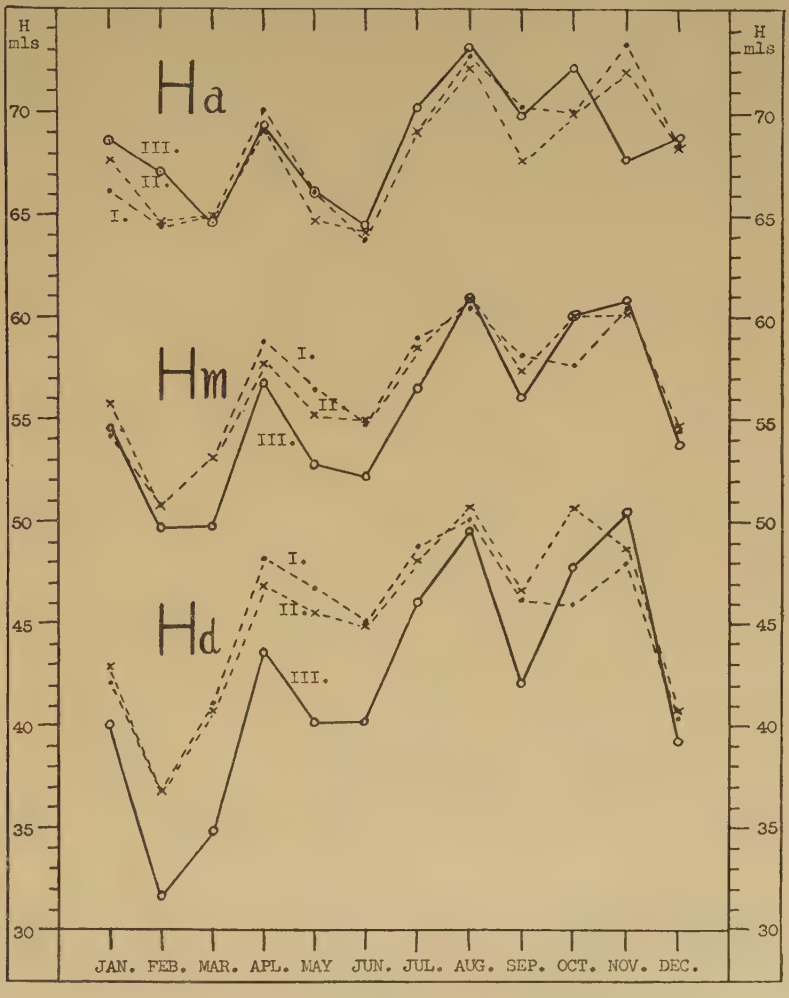


FIG. 2.—*Influence of Annual Showers on Meteor Heights.*

Finally it was decided to exclude from the computations all constituents of the following annual meteor showers :—

Date	Name	Radiant	
		°	°
Jan. 2–3	Quadrantids	232	+52
Apr. 20–21	Lyrids	270	+34
May 1–7	Eta Aquarids	336	0
July–Aug.	Delta Aquarids	345	−17
,,	Perseids	44	+57
Oct. 15–25	Orionids	90	+15
Nov. 15	Leonids	151	+22
Dec. 10–12	Geminids	106	+32

* *Astron. Abhandlungen*, 4, nr. 5. † *Harvard Coll. Obs. Circular* 391, 1934.

The exclusion of the individuals belonging to the above streams left a total of 2113 meteors available for further investigation. As radiant points were not always indicated in the original publications, possibly a few members of these streams remain in the data.

The statistics of the shower meteors are given in Table II, where the respective columns for each point in the meteors' paths give the total number of meteors, the mean height, the amount by which this height exceeds the mean height of sporadic meteors in the same month (taken from Table V), and finally the ratio: Shower height/Sporadic height. The δ Aquarids and Perseids, occurring in both July and August, are given separately for each month as well as in combination.

TABLE II

Statistics of Shower Meteors

	Ha				*Hm*				*Hd*			
	No.	*H*	Δ*H*	Sh./Sp.	No.	*H*	Δ*H*	Sh./Sp.	No.	*H*	Δ*H*	Sh./Sp.
		m	m			m	m			m	m	
Quadrantids	32	68·2	−1·0	0·97	32	58·3	5·8	1·11	32	49·8	13·0	1·35
Lyrids	20	76·2	8·2	1·12	20	62·7	7·6	1·14	21	53·1	10·9	1·26
η Aquarids	5	62·6	−3·6	0·95	5	57·2	4·8	1·09	5	51·8	12·3	1·31
δ Aquarids :												
July	9	54·0	−16·3	...	9	47·2	−9·2	...	9	40·0	−5·7	...
August	6	68·3	−2·3	...	6	60·4	1·5	...	6	52·2	3·8	...
All	15	59·7	−10·7	0·85	15	52·3	−4·7	0·92	15	44·8	−2·2	0·95
Perseids :												
July	18	78·4	8·1	...	18	65·3	8·9	...	18	55·2	9·5	...
August	172	79·4	8·8	...	172	52·6	4·2	...	171	65·8	6·9	...
All	190	79·1	8·7	1·12	189	65·8	8·8	1·15	190	52·5	5·5	1·12
Orionids	33	80·6	9·7	1·14	32	68·8	10·0	1·17	32	59·2	13·0	1·28
Leonids	311	77·5	13·6	1·21	308	64·4	7·0	1·12	326	59·1	12·8	1·28
Geminids	18	75·3	8·1	1·11	18	58·4	4·5	1·12	18	41·5	1·3	1·07
Average	624	76·0	4·1	1·06	623	64·0	5·5	1·10	639	53·9	8·3	1·20

In the first place it must be pointed out that the data given in the above table cannot be regarded as a reliable indication of the statistics of these showers, except in the case of the Perseids and Leonids, owing to the small number of meteors from which the averages are derived. The figures for the two Aquarid showers are especially unreliable because meteors from a southern shower seen by an observer in the northern hemisphere would not be a true sample of the individuals of that shower.

The third column in each set reveals the fact that these shower meteors, excluding the two Aquarid showers, experience their luminous careers from 5 to 14 miles higher in the atmosphere, both at apparition and disappearance, than does the normal, non-shower meteor. The Quadrantid shower, it may be noted, is the only one in the northern hemisphere in which all three points are not higher than the average sporadic meteor.

The ratio Shower/Sporadic shown in the fourth column of each set,

averaging 1·12 for the three sets, is slightly less than that found by Öpik, whereas, as Öpik points out, even the value he found is below what might be expected from a consideration of the relative velocities of the two types of meteors. As I have already mentioned, direct comparison between Öpik's figures and my own in many cases would be misleading, and that would be so in this instance, for my shower meteors are the constituents of the rich annual showers, while Öpik's are those emanating from observed "group radiants." It is interesting to note the close agreement between the various ratios in column four of the table, this agreement being particularly well marked in the case of *Hm.*

An investigation of the respective ranges of the two types of meteor from the present data reveals that the shower meteors, far from being in excess of the sporadic range, as indicated in Öpik's research, have a range amounting to only 0·82 of the sporadic range. The figures on which this conclusion is reached are given in Table III.

TABLE III

Ranges of Meteor Paths

	Range		Shower Excess	Ratio Sh./Sp.
	Shower	Sporadic		
	m	m	m	
Quadrantids	18·4	32·4	−14·0	0·57
Lyrids	32·1	25·8	− 2·7	0·90
η Aquarids	10·8	26·7	−15·9	0·40
δ Aquarids	14·9	24·6	− 9·7	0·60
Perseids	26·6	24·5	2·2	1·09
Orionids	21·4	24·7	− 3·3	0·87
Leonids	18·4	17·6	0·8	1·05
Geminids	33·8	29·0	4·8	1·17
Average	20·7		− 4·7	0·82

The data reveal that the algebraic mean of the excess (Shower minus Sporadic) range (*Ha* − *Hd*) in column four is −4·7 miles, leaving the shower meteor only 0·82 of the normal range.

The analyses of meteor spectra made by Millman * indicate that 100 per cent. of the shower meteors studied were mainly or exclusively stony, while 64 per cent. of the sporadic meteors were irons. If this is true of all meteors, composition may be the factor causing the observed greater heights of shower meteors, for the stones could conceivably be volatilised at greater heights than the irons.

On the other hand, the slower-moving shower meteors should not become luminous as high as the faster sporadic meteors. The velocity factor, therefore, working in the opposite direction, may offset the composition factor and create a stalemate.

At this stage in my investigations I made an interesting discovery. In

* *Harvard Annals*, **82**, Nos. 6 and 7, 1937.

Table IV I give the mean heights of Perseid meteors for all groups of data arranged according to date. The figures are depicted graphically in fig. 3, where the straight lines at 70·6 miles, 58·9 miles and 48·4 miles are the mean heights of the non-shower meteors for August for comparison. The curve of Perseid activity for the years 1920–21, taken from Öpik's double count estimations *, is also given, the scale for this appearing on the right of the diagram.

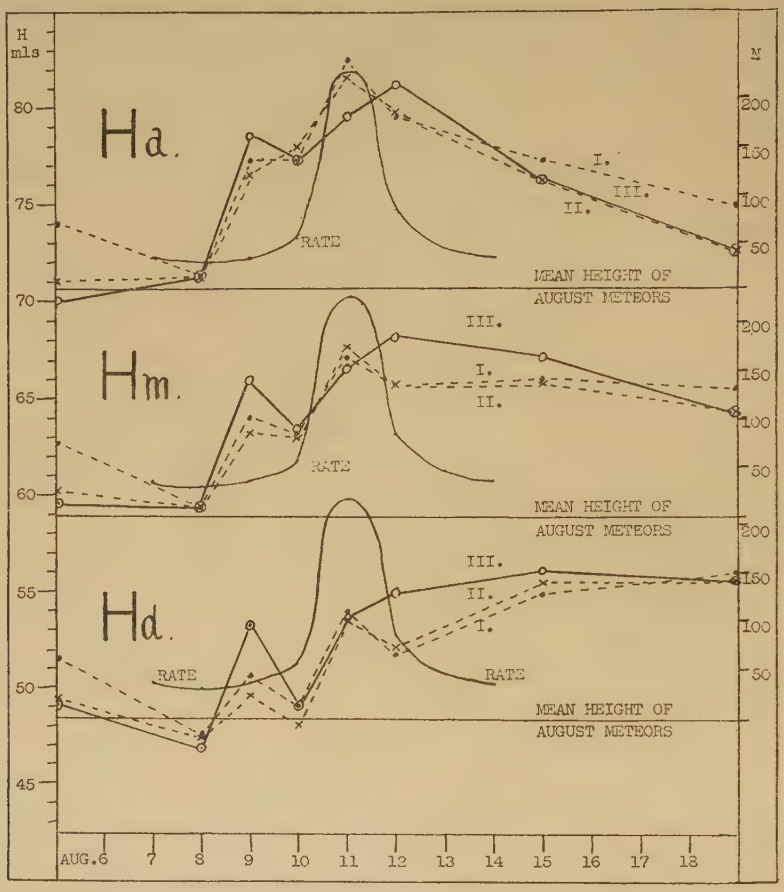


FIG. 3.—*Relation between Height and Numbers of Perseid Meteors.*

An inspection of fig. 3 makes it evident that a few days prior to the maximum of the Perseid shower the mean heights of the meteors are little greater than those of the normal, sporadic meteors. With increasing numbers of meteors, however, the mean heights increase to a maximum coinciding with the visual maximum on August 11. Thereafter, the mean heights slowly decline.

Unfortunately, the other annual showers are not suitable for a similar analysis, only the Perseids providing a large number of meteor heights spread over a considerable period of activity.

* *Pubs. Tartu Obs.*, 25, Nos. 1 and 4.

I must content myself at this stage with mentioning a number of possibilities indicated by this phenomenon. In the first place the perturbative influence of the Earth's many passages through the Perseid meteor stream may have resulted in the more volatile meteoroids constituting it being concentrated in the central core, leaving the remainder to form the very wide fringes the stream is known to possess.

TABLE IV

Heights of Perseid Meteors

Date	Group	*Ha*		*Hm*		*Hd*	
		No.	m	No.	m	No.	m
Aug. 3–7	I	11	74·0	11	62·7	11	51·5
	II	13	71·0	13	60·2	13	49·4
	III	14	70·0	14	59·6	14	49·1
Aug. 8	I	4	71·3	4	59·4	4	47·5
	II	4	71·3	4	59·4	4	47·5
	III	5	74·2	5	60·5	5	46·8
Aug. 9	I	5	77·4	5	64·0	5	50·6
	II	6	76·6	6	63·2	6	49·6
	III	9	78·6	9	66·0	9	53·3
Aug. 10	I	25	77·4	25	63·2	25	49·0
	II	37	78·0	37	63·0	37	48·0
	III	45	77·3	45	63·3	45	49·1
Aug. 11	I	32	82·5	32	67·7	32	54·0
	II	41	81·6	41	67·6	41	53·5
	III	50	79·5	50	66·6	50	53·6
Aug. 12	I	18	79·5	18	65·7	18	51·8
	II	21	79·6	21	65·9	21	52·1
	III	36	81·4	36	68·2	36	55·0
Aug. 13–17	I	13	77·4	13	66·2	13	54·9
	II	19	76·1	19	65·8	19	55·5
	III	27	76·1	27	67·2	27	56·1
Aug. 18–22	I	1	75·0	1	65·5	1	56·0
	II	2	72·5	2	64·3	2	55·5
	III	2	72·5	2	64·3	2	55·5

On the other hand, the constitutions and densities of the meteoroids may not vary through a cross-section of the stream. In that case an explanation of the phenomenon must be looked for in the atmosphere itself, from the effects of the mass bombardment by a greatly increased number of meteors at the times of annual showers.

If the second alternative is correct, there are several possible explanations which might fittingly be tested by specialists in the various sciences concerned. There is a possibility that meteor showers have a mass-ionisation effect similar to that of a chromospheric eruption and that the ionisation

built up may persist for some time. The meteorologist may be able to find that an increase of density arises from the energy expended in impact or an increase in temperature from the effect of solar radiation upon the abnormal

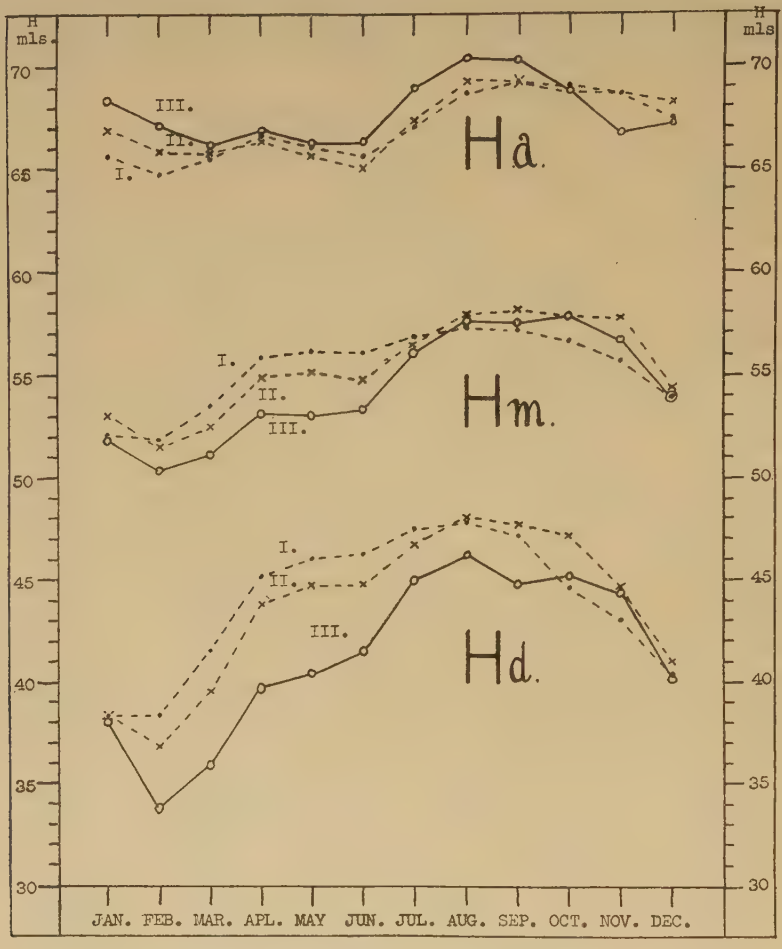


FIG. 4.—*Seasonal Variation in Meteor Heights.*

quantities of meteoric dust then present. Likewise, the chemist may find the phenomenon due to dissociation in the atmosphere.

One additional point may be mentioned at this stage. If the fringes of a meteor stream differ materially from its core, sporadic meteors will not reveal any increase in height at the time of an annual shower. If, however, it is the increased meteoric activity which raises the heights of the shower meteors concerned one would expect a similar, although smaller, effect on the sporadic meteors occurring at that time. Taking the three-day period August 10–12 about the Perseid maximum we do, in fact, find the mean heights of sporadic meteors to be 1·0 miles above the point of appearance of all August non-shower meteors, but at the point of disappearance the

opposite is found, the sporadic meteors in the period selected ending 0·8 miles below the mean for the month.

We must not attach too much weight to this test. In the first place the mean heights are based on only 159 meteors, and sporadic meteors exhibit a greater range of heights than shower meteors, tending with such small numbers to mask any effect. Then again the "bombardment" effect may be confined to that portion of the atmosphere where the shower meteors occur most frequently, while the sporadic meteors occur in all parts of the sky and at all hours of the night.

Seasonal Variation in Height.—Table V gives the monthly average heights of non-shower meteors for the three groups of observers. The arithmetic mean heights are given for the three points, *Ha*, *Hm* and *Hd*, together with the number of meteors upon which the averages are based.

The data contained in Table V have been smoothed by the formula $x_s = \frac{1}{4}(x_{r-1} + 2x_r + x_{r+1})$ to present the seasonal variation graphically in fig. 4. As the division into months is an artificial one, this smoothing tends to counteract the effect of an abnormal meteor occurring very early or very late in a month and is not as severe a smoothing as most other methods.

An inspection of the graph in fig. 4 reveals several interesting points. The seasonal variation is so well marked in all three points of the path as to be unmistakable, the minimum occurring in February and the maximum in August–September. The longer range of the meteors in Group III, the point of appearance being above and of disappearance below the other groups, arises from the fact that the Hoffmeister-von Niessl catalogue, as its title implies, concerns itself only with great meteors, and therefore yields greater ranges than does the more nearly normal British material. The other foreign material, being mainly confined to the months of August and November, has no general effect on the curves.

Inspection of the figure reveals a significant variation in the amplitude of the curves, greater in the lower levels and decreasing progressively as the higher altitudes are studied. The variation is shown in Table VI.

Finally, the maxima and minima show some indication that they occur later in the higher altitudes. The summer maximum (Group III) occurs in August for the lowest point (45 miles altitude), in September for mid-point (58 miles altitude) and in September for the highest point (70 miles altitude). Similarly, the minima for the respective points occur in February, February–March and April.

In his paper Öpik indicates three causes which might give rise to the observed variation in *Hc*: (1) a seasonal change in the height of the atmosphere, (2) a seasonal change in the mean velocity of meteors, or (3) a seasonal change in the composition of meteors. From a consideration of the various points brought out in the graph it seems that the second and third alternatives may be ruled out, as they would have no influence on the varying amplitudes exhibited in the curves nor cause the maxima and minima to occur later in higher altitudes. For both these reasons the cause seems to lie in the heating and expansion of the atmosphere itself.

TABLE V

Mean Monthly Heights of Non-shower Meteors

Month	Group	*Ha*		*Hm*		*Hd*	
		No.	m	No.	m	No.	m
January	I	43	65·1	43	51·9	43	38·6
	II	65	65·9	65	53·7	65	39·0
	III	91	69·2	91	52·5	97	36·8
February	I	31	64·5	30	50·8	32	36·8
	II	47	66·6	46	50·4	47	34·7
	III	71	67·1	70	49·7	73	31·6
March	I	54	65·0	54	53·1	54	41·2
	II	65	64·1	65	51·3	65	38·6
	III	88	64·8	88	49·8	89	34·9
April	I	88	67·4	89	57·1	89	46·7
	II	107	68·1	108	56·8	108	46·1
	III	132	68·0	133	55·1	138	42·2
May	I	43	66·6	43	56·4	43	46·1
	II	64	65·4	64	54·8	64	44·5
	III	90	66·2	90	52·4	90	39·5
June	I	42	63·9	42	54·8	43	45·1
	II	54	63·5	54	53·9	56	43·8
	III	79	64·6	79	52·2	86	40·3
July	I	88	68·0	88	58·2	89	48·3
	II	121	67·6	121	56·5	126	46·6
	III	190	70·3	189	56·4	204	45·7
August	I	122	68·2	123	56·4	127	48·3
	II	315	70·4	319	58·8	337	49·8
	III	432	70·6	433	58·9	468	48·4
September	I	99	70·3	99	58·2	98	46·2
	II	166	68·8	166	57·2	166	45·8
	III	200	69·9	200	56·1	209	42·2
October	I	64	68·4	64	55·7	65	43·6
	II	169	69·0	169	59·0	170	49·2
	III	211	70·9	210	58·8	220	46·2
November	I	91	69·4	91	56·7	92	44·5
	II	157	68·5	158	55·8	157	44·3
	III	412	63·9	397	57·4	434	46·3
December	I	52	67·7	52	53·7	52	39·3
	II	83	69·2	83	54·1	83	40·3
	III	110	67·9	110	52·8	113	38·9

TABLE VI

Amplitudes of Curves

	Ha	*Hm*	*Hd*
Group I :	m	m	m
Maximum *H*	69·4	57·3	47·8
Minimum *H*	64·8	51·8	38·4
Amplitude	4·6	6·5	9·4
Per cent. of *H*	6·8	11·8	21·8
Group II :			
Maximum *H*	69·3	58·0	48·1
Minimum *H*	65·8	51·4	36·8
Amplitude	3·5	7·6	11·3
Per cent. of *H*	5·2	13·7	26·5
Group III :			
Maximum *H*	70·0	55·7	45·3
Minimum *H*	67·2	51·2	30·2
Amplitude	2·8	4·5	15·1
Per cent. of *H*	4·1	8·4	40·0

Variation in Width of Meteor Zone.—A marked seasonal variation in the width of the zone in which meteors occur is evident when the monthly ranges *Ha–Hd* are determined. The variation is shown in Table VII, the zone being widest in winter and having its least width in summer.

TABLE VII

Width of Meteor Zone

Month	Group I	II	III	Month	Group I	II	III
	m	m	m		m	m	m
Jan.	27·3	28·6	30·4	July	19·5	20·3	24·0
Feb.	26·4	29·0	33·4	Aug.	20·9	21·3	24·2
March	24·0	26·2	30·3	Sept.	23·2	21·6	25·5
April	21·4	22·6	27·1	Oct.	24·6	21·7	23·7
May	20·1	20·9	23·9	Nov.	25·7	24·3	22·3
June	19·4	20·3	24·9	Dec.	27·1	27·2	27·0

Comparison with the Harvard Results.—The question naturally arises, How does the seasonal variation shown in this paper compare with that found by Öpik from the Harvard data? Direct comparison is of course no criterion, because the Harvard means are harmonic and mine arithmetic. For purposes of comparison I have therefore computed the harmonic mean heights for all the British data, the figures being given in Table VIII.

In spite of the fact that the harmonic means do not yield a true picture of the data used, good seasonal curves are again revealed when the figures are smoothed as before to provide fig. 5. For obvious reasons the harmonic means are slightly lower than the arithmetic ones. For purposes of comparison Harvard's heights are indicated by triangles, those for *Ha* and *Hd*

being white and those for *Hc* (my *Hm*) being black to avoid confusion. The figure reveals that there is close agreement between Öpik's results and mine for the point for which he demonstrated seasonal variation (*Hc*). His curves for *Ha* and *Hd*, however, exhibit only about half the range shown by mine, which possibly arises from the manner in which the data were gathered.

TABLE VIII

Harmonic Mean Heights of Meteors

Month	Group	*Ha*		*Hm*		*Hd*	
		No.	m	No.	m	No.	m
January	I	43	64·2	43	50·7	43	32·9
	II	65	64·0	65	50·0	65	32·6
February	I	31	62·4	31	50·3	32	32·7
	II	44	61·6	44	47·8	45	28·7
March	I	52	63·4	52	51·7	52	36·5
	II	63	59·3	63	46·4	63	29·2
April	I	84	65·5	84	55·7	84	41·6
	II	103	64·9	103	54·6	103	40·4
May	I	47	64·9	47	54·9	47	41·9
	II	63	64·2	63	54·1	63	39·8
June	I	42	62·7	42	53·4	43	40·9
	II	52	62·6	52	53·2	53	40·9
July	I	91	65·9	91	56·0	92	43·8
	II	116	64·1	116	54·0	118	38·5
August	I	120	63·9	120	53·9	120	40·9
	II	288	64·2	288	55·0	292	41·6
September	I	97	67·1	97	55·3	97	41·3
	II	161	65·2	160	54·5	160	40·7
October	I	71	66·2	71	53·3	70	40·7
	II	165	65·5	165	55·3	165	43·7
November	I	90	66·3	90	54·1	90	39·4
	II	154	65·4	154	53·7	154	38·7
December	I	52	65·3	52	51·1	51	34·7
	II	77	65·5	77	51·5	75	35·5

Taking only the central curve of *Hm* (*Hc*), it is significant to note the close agreement between Öpik's curve and mine in the months where my data contain the greatest number of meteors. Near the minimum period of December to March the harmonic curves fall much further below the arithmetic curves than they do in the other months because in the sparse months a few low end-points show up in the harmonic means with all their

weight. In the remainder of the year the curve is nearer the true state of affairs because the larger number of normal meteors has to a greater extent counteracted this tendency.

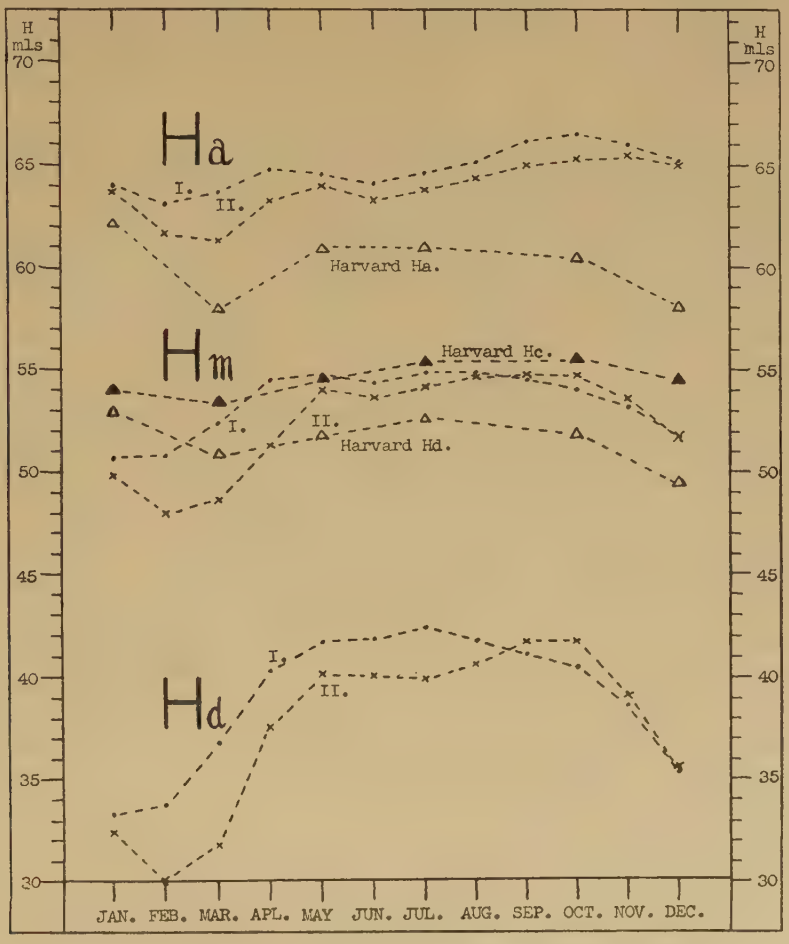


FIG. 5.—*Comparison of Harmonic Mean Heights with Harvard's.*

An inspection of fig. 5 reveals that in the first months of the year the minimum is considerably below that shown by Öpik's smoothed curve (*Hc*) corrected to sea-level, as the following figures show :—

	Jan.	Feb.	
	m	m	
Harvard *Hc*	53·8	53·3	Average difference 4·65 miles.
My *Hm*	50·0	47·8	
Difference	3·8	5·5	

The amplitude of my harmonic mean curve is therefore much greater than Öpik's, making the seasonal trend appear more pronounced. The few low-ending meteors in those months, however, have exaggerated the

curves, which is clearly shown by removing all meteors whose end-points come below 20 miles, of which there were 13 in a total of 109. The divergence between the two curves in those months then becomes :

	Jan.	Feb.	
	m	m	
Harvard *Hc*	53·8	53·3	Average difference 2·65 miles.
My *Hm*	51·7	50·1	
Difference	2·1	3·2	

The divergence between Öpik's curve and mine in January and February would therefore be almost halved by the removal of these few low end-points, which shows in another way that the use of the harmonic means in this paper would have been misleading. It may be pointed out also that in the above figures I have dealt only with *Hm* divergences and low end-points have only half the weight in the *Hm* curve that they would have in *Hd*.

Accuracy of Data Used.—As an amateur with limited time available for further work on a research which has already taken up considerably more time than I can well afford, I have spared myself the labour of computing the probable errors of the 108 points involved in the curves deduced. It may suffice to indicate that the spread of the data in the frequency curves (with the wings removed, following Öpik's precedent) is very little greater than the spread in Harvard's own frequency curves, and my data therefore may be taken as comparable in accuracy with those of Harvard.

The fact that a seasonal trend is revealed in one point of the meteors' apparent paths determined from the data of one computer might be shown with greatly increased material to be accidental. When that trend appears equally well in all three points considered the probability of the effect being real is increased. When to those data are added other data independently computed by other workers, more than doubling the available material, without affecting the seasonal trend, strong support is given to the conclusion that the variation revealed must be real. Finally, the fact that my curve for *Hm*, based on 2000 meteors, agrees within an average of 2·1 miles with a similar curve found by Öpik from an independent set of 3000 meteors places the matter beyond reasonable doubt.

The exhaustive investigation made by Öpik into the various errors inherent in meteor observing has created a tendency, exhibited in recent papers, to belittle the value of visual meteor observations. One not familiar with the practical side of meteoric astronomy might even be inclined to doubt the advisability of employing data such as I have used in the present paper.

The Harvard data concerning errors of observation apply only to their own particular methods and are not indicative of the errors in other methods for several reasons. In the first place, Harvard did not employ experienced meteor observers, nor did they observe in the usual manner, reticules being used instead of stars for reference purposes, and all who have used both methods will agree that the reticule is inferior. In the reductions of observations new methods were also employed.

Consequently, Harvard occasionally errs in ways not common to visual observers. Olivier has pointed out * how Harvard has missed rich annual showers on nights when they must have been present, how group radiants deduced embrace a greater spread of paths than is permissible in visual work, how successive radiants of annual showers show motion in a direction opposed both to theory and observation.

In spite of these differences in method, Öpik implies that all visual meteor observations are less reliable than the Harvard results, and asserts that any mean heights deduced by arithmetical means contain a positive systematic error proportional to the square of the observational error, which he apparently considers to be large. He assumes that the amount of the error is indicated by the difference between the arithmetic and harmonic means. He suggests, for example, that because the harmonic mean of a series of Leonid heights deduced by the American Meteor Society was 12·7 km. below the published arithmetic mean of 130·1 km. the mean was therefore 11 per cent. in error. The frequency curve of the American Meteor Society's data, being based upon only 43 heights, cannot be a true indication of all the society's data upon Leonids, but in this particular case it reveals a marked negative skewness, and the use of the harmonic mean, instead of tending to counteract the skewness, as it did in Harvard's own data (where the skewness was positive), actually exaggerates the position. This is the real reason why the harmonic mean falls so far below the arithmetic in the sample under discussion, and is not, as Öpik implies, an indication of large observational errors in the data.

With regard to the accuracy of the material published by Denning, that computer himself has defined the probable accuracy in good cases as being within a mile of the truth.† Lindemann and Dobson have confirmed this ‡ by pointing to the relatively sharp limits to the region above the ozone layer in the frequency curve for *Hd* (see fig. 1), which is an indication of the small probable error in the estimates of height by eye observation. If the errors were at all large, this narrow region of discontinuity would not be sharply observable. They therefore infer that the probable errors in heights obtained by the best observers must be very small, probably not more than a few kilometres. The accuracy of the work of the British Astronomical Association's Meteor Section, which provides a portion of my data, has been demonstrated in recent papers and requires no additional comment here. §

The best criterion, in my opinion, of the accuracy of the real paths of the various workers which provide the data upon which the present paper is based, is that they exhibit the seasonal curve not only in the middle point of the path, as did Harvard's, but also show it at beginning- and end-points as well. This is sufficient answer to any suggestion that visual work such as has been carried on in Great Britain for over half a century is subject to grave errors and cannot be relied upon.

* *Pop. Ast.*, **46**, 6, 325, 1938.
† *M.N.*, **72**, 395, 1913.
‡ *Proc. Royal Soc.*, A, **102**, 411, 1923.
§ *Journ. B.A.A.*, **48**, 337 ; **49**, 113, 1938 ; **49**, 146, 1939.

In conclusion I must acknowledge my indebtedness to Dr. L. V. Berkner, Dr. E. Kidson and Professor Burbidge for their interest and assistance, and especially to Dr. K. E. Bullen for invaluable advice in some difficult problems connected with the preparation of this paper.

Summary

1. All available published real paths of meteors, both British and foreign, were used in an attempt to confirm the seasonal variation found by Öpik in the centre of trails observed by the Harvard Meteor Expedition.

2. Öpik having asserted that harmonic mean heights should be used in discussions of heights, as the arithmetic means were subject to errors of observation, the data were examined from a statistical viewpoint and it is shown that in the present instance it is correct to use the arithmetic means. Examples of errors arising from the use of harmonic means are given.

3. It was found necessary to exclude shower meteors from the data, as they began and ended up to 14 miles higher than the sporadic meteors. Some possible explanations are indicated.

4. The slightly greater range of heights found by Öpik in shower meteors is not confirmed, the average shower meteor in my data having a shorter range than the sporadic.

5. The data, after excluding the shower meteors, reveal a well-marked seasonal variation in the three points examined. The amplitude of the variation is greatest in the lower levels of the atmosphere, where maxima and minima appear to occur earlier than in higher levels. These facts indicate that the cause of the variation lies in the atmosphere itself.

6. A similarly well-marked seasonal variation is found in the width of the meteor zone.

7. For direct comparison with the Harvard data the harmonic mean heights are also given, and, while there is close agreement between their results and mine, several weaknesses in the use of harmonic means are exposed.

8. In a consideration of the accuracy of the visual meteor heights used in this paper the high standard of accuracy and the value of this material are defended.

1 *Melford Street,*
Auckland, W. 1,
New Zealand :
1939 *April* 11.

THE FOG EFFECT IN PHOTOGRAPHIC ASTRO- AND SPECTRO-PHOTOMETRY

G. Alter, D. R. Barber and D. L. Edwards

1. *Introduction*

It has been known for some time that if two, or more, photographs of star fields are taken consecutively on the same plate (with a shift of the plate between exposures) then the star images produced by the earlier exposures are of greater diameter (or density) than the later ones. This will evidently have an influence on accurate measurements in photographic photometry, and also (since the effect holds for spectra as well as stellar images) in spectrophotometry. In the latter case even a single star spectrum could be affected if it were broadened by trailing once on the plate. There would be differences of intensity across the width of the spectrum, since different parts of the trail correspond to different exposure "epochs."

It is therefore of importance to investigate the nature of this effect, and the conditions under which it operates. The origin of the present investigation arose in the course of work on colour-temperatures, involving the comparison of two stellar spectra on one plate which also received a certain amount of general fogging from the sky background (due to the use of an objective prism). It was desired to ascertain if any difference existed between the effects of a fogging exposure before, or after, the spectrum exposure. In the colour-temperature plate mentioned, the second spectrum would fall on a pre-fogged background due to the first exposure, whilst the second exposure would itself add post-fog to the first spectrum. The results indicated a real, and significant, difference, so that the order in which the two spectra were taken was of importance. An independent discovery was thus made of the above-mentioned effect (or something similar) as applied to stellar spectra. This was later confirmed for the case of star diameters.

The present paper is intended to draw attention to the existence, and importance, of this effect; to show that it is due to the different behaviour of pre- and post-fogging background light, and not—as suggested by earlier investigators—to humidity producing a decrease in emulsion sensitivity between the two exposures (section 3); to indicate the nature and the probable magnitude of its influence on photographic measurements in (*a*) stellar photometry (section 5), and (*b*) spectrophotometry (section 6); finally, to study some of the different factors which influence the effect —such as the type of photographic emulsion, fogging intensity, wavelength, etc.

The paper is chiefly a continuation, and expansion, of a note on the same subject recently published by two of us.* It includes a short discussion on the theoretical basis of the effect, and confirmation by laboratory experiment (section 7).

* *The Observatory*, **62**, 327, 1939.

2. *Early Work*

In the investigation of RR Draconis F. H. Seares * quotes from a personal letter of Professor Pickering that the magnitude effect between the first and the last image "is about a quarter of a magnitude", whilst Seares' plates "are free from the photographic effect." The last column of Seares' Table VII, however, suggests the presence of a small effect, since the apparent magnitudes of the first images are too bright and of the last too faint. Greenwich plates † also have "no systematic difference in the images of stars . . . obtained by the first and last exposures." In a later paper ‡ Seares mentions the effect on some earlier Mount Wilson photographs, but "it does not appear to be appreciable upon others subsequently obtained. . . . The cause is obscure, though apparently photographic in origin."

The importance of the effect in photometric work was met in the standardizing of the North Polar Sequence (NPS). In her comprehensive work Miss Leavitt § draws attention to the fact that on double image plates the second image appears fainter. "Apparently some cause, or combination of causes, tends to make the later exposure fainter." In a careful investigation of the difference between the results of the Harvard and the Mount Wilson sequence measurements Seares ‖ suggests, "that the major part of the change should occur within a comparatively short interval after the beginning of exposure." This is, however, contradictory to an earlier assumption ¶ "that the phenomenon develops proportionally to the time, which in the absence of more definite information is a perfectly natural supposition", and to the similar treatment of this phenomenon by Miss Leavitt.** A new recognition of the effect in photometric work occurred in exposures of Selected Areas, by P. J. van Rhijn ††; "the effect is probably due to a decrease of the sensitivity of the plate during the first exposure caused by moisture in the atmosphere."

A similar effect was found by W. Hartree and A. V. Hill ‡‡ on bromide paper used for galvanometer tracings and exposed at the same time to weak light, the earlier (post-fogged) parts of the tracings being denser than the later. In a recent paper J. H. Webb and C. H. Evans §§ (see section 7) studied the stability of latent-image silver specks, and found the same effect due to post-fogging. It may be mentioned that nearly 40 years ago Edward S. King ‖‖ investigated the influence of post-fogging on plates in the laboratory. It "was intended to show if it makes any difference whether a plate is fogged before, or after, exposure to the light which is to be measured." His conditions are not quite comparable with those of normal stellar working, but even in his case the difference between the two exposures is positive (about $0^m.02$).

* *Ap. J.*, **36**, 374, 1912.
† *M.N.*, **74**, 40, 1913.
‡ *Ap. J.*, **39**, 311, 1913.
§ *H.A.*, **71**, 121, 1914.
‖ *Ap. J.*, **41**, 288, 1915.
¶ *Ap. J.*, **41**, 284, 1915.
** *H.A.*, **71**, 124, 1914.
†† *Trans. I.A.U.*, **6**, 227, 1938.
‡‡ *J. Sci. Instr.*, **9**, 329, 1932.
§§ *Comm. No.* 687 from Kodak Research Labs., 1938.
‖‖ *H.A.*, **59**, 17, 1901.

In the early stages of photography, 1841, post-fogging * was used to improve prints from contrasty negatives. It was recognized and investigated by Becquerel on Daguerrotype plates†, and later it was found that the violet part of the spectrum was more effective than the red. Schulz-Sellack explained the "continuing" effect not as a peculiarity, but as a consequence of the necessary first exposure. As another application X-ray plates were post-fogged ‡ after exposure by diffuse light under a ruby red glass, and they appeared as if radiated 3 to 4 times as long. In a study on the Herschel effect W. Leszynski § investigates the influence of post- and pre-fogging of emulsions by red light.

3. *The Effect of Humidity*

One of the first questions in the present investigations was to separate a possible humidity effect.

Experiments on this effect, stimulated by the above-mentioned remark of Pickering, were made by Mees ‖, later by J. Cuffey ¶ and G. Z. Dimitroff **, but as these were under conditions very different from those of normal astronomical work the results are not directly applicable to the present investigation. The three papers compare images on different plates treated under different humidity conditions, whilst the present question includes in addition the change by humidity of two or more images on one plate. Two results, however, are important for this paper: (*a*) the fact that a decrease of sensitivity with increased humidity is established; and (*b*) that saturation is reached in about 2 hours, after which no further change takes place in the emulsion.

The present method of determining this effect was governed by the normal conditions obtaining in astronomical photography. The problem was to determine the difference between the change (if any) of a latent image, due to the influence of humidity, and the effect on a second image of the plate's loss of sensitivity before exposure. The method, described in section 5, compares the image diameters of two NPS exposures on one plate, of equal exposure time, taken with the Mond Astrograph. Three kinds of investigation were used:

(*a*) One plate was exposed for 14^s only (to avoid a possible fog effect) to the NPS, the cap of the telescope was afterwards closed and covered by a black cloth to exclude any light, whilst the shutter of the plate-carrier remained open to subject the plate to the air humidity for an interval of 2 hours; finally the plate was again exposed to the NPS for 14^s, after a shift in declination. A hazy night was chosen, without moonlight, to have a fairly humid air without fogging background. The emulsion was Ilford Hypersensitive Panchromatic (HSP), securing high sensitiveness and a large spectral range. The average difference between the diameters of the first

* J. M. Eder, *Ausf. Hdb. d. Phot.* (3rd ed.), **2**, 326, 1927.
† *Ibid.* (1st ed.), **1**, 313, 1891.
‡ *Ibid.* (3rd ed.), **2**, 319, 1927.
§ *Z. f. wiss. Phot.*, **14**, 275, 1926.
‖ *Ap. J.*, **40**, 236, 1914.
¶ *H.B.*, 905, 1937.
** *H.C.*, 430, 1938.

and second images, expressed in magnitudes, was $+0^{m}.02$, against zero for a plate taken under the same exposure conditions, but without subjection to humid air (see section 5).

(*b*) Another plate was first exposed for 1^{h} to the air in the telescope by closing the cap and opening the dark slide, then it was exposed for 14^{s} to the NPS, then again subjected to the air for another 1^{h} under the same conditions, and, finally, exposed a second time to NPS for 14^{s}. The night was moonless and clear, to get the conditions of a normal working night. The arrangement is the same as usual in the cluster work on the Mond Astrograph *, except the exposure time which has been, of course, cut down to avoid fog effect. The emulsion was again HSP. The average difference in magnitudes was $0^{m}.03$.

(*c*) To show the existence of a true fog effect a third experiment was made. A plate was exposed on a very hazy day for $5^{h}\ 26^{m}$ to the air in the telescope. Afterwards it was exposed twice for $3^{m}\ 30^{s}$ to the NPS, with only an interval of 30^{s} between the two exposures. In this way a differential humidity effect was avoided, and an apparent effect could be due only to the fog influence. The night was hazy, moonlight. The average difference between the magnitudes of the two images was $0^{m}.18$, which agrees quite well with the effect under normal conditions (see section 5).

These experiments make it clear that a humidity effect in a normal working night, even when it is hazy and a very sensitive plate is exposed for 2^{h} to the air, does not exceed $0^{m}.03$. It is considerably lower than the normal fog effect and is therefore negligible.

(*d*) Further proof that it can be neglected was found in spectroscopic work. The effect, if present, should be visible when a plate is exposed to a single-trailed spectrum on a night without moon, when it would give rise to a decrease in density from beginning to ending of the trail. Plates with a fogged background show an effect, but those without it do not.

4. *Definition*

The greatest part of the general effect under discussion is thus due to fogging background light. The first image of a double exposure (or the first edge of a trailed spectrum) is increased in diameter (or density) by post-fogging light, while the second is increased by pre-fog. These two increases, however, are not equal, the former being larger than the latter. In this paper the difference between the two increases is defined as the "fog effect," and is denoted by the symbol *F*.

5. *The Fog Effect in Stellar Photometry*

In stellar photometry the present investigation was limited to focal star images. The stars photographed were those of the NPS only, to avoid altitude differences. The displacement of the two pictures (of equal exposure time) was obtained by the slow movement in declination, and was

* *M.N.*, **100**, No. 5, 1940.

only about 1 mm. to avoid sensitivity differences in the emulsion. The magnitude determination is described in the cluster work of one of us.* Five to 15 stars were measured on every plate, according to the number of suitable stars given by different exposure times, to get a reliable mean of the difference between the two image diameters. The investigation was restricted to three emulsions, namely, Ilford Hypersensitive Panchromatic (HSP), Ilford Double X-Press (DXP) and Barnet Super-Press (BSP). The sets of exposures which required unchanged atmospheric and back-

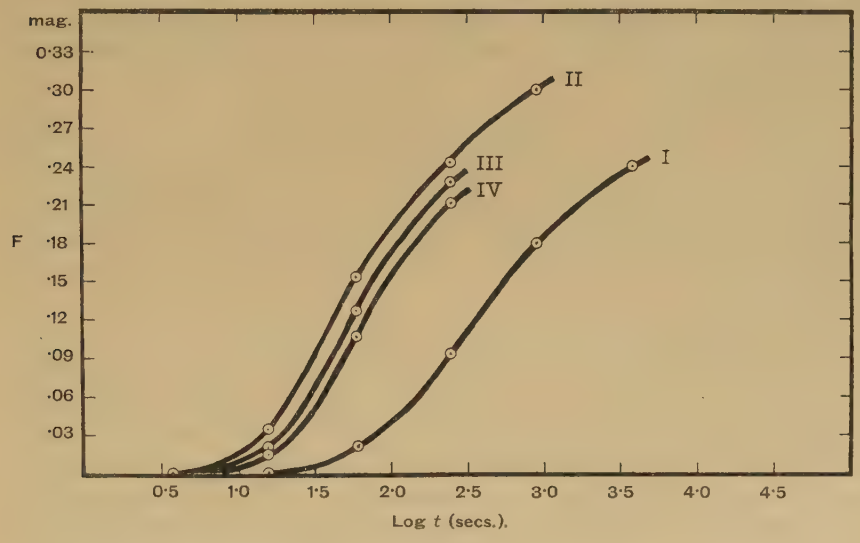


FIG. 1.—*Increase of fog effect with exposure time for different emulsions.*

(*I*) *H.S.P. no moon.* (*III*) *D.X.P. full moon.*
(*II*) „ *full moon.* (*IV*) *B.S.P.* „ „

ground conditions were taken on the same night. All exposures were taken on camera A of the Mond Astrograph.

The four principal questions were the dependence of the fog effect on (*a*) the total exposure time, (*b*) the fog intensity, (*c*) the kind of emulsion used and (*d*) the star magnitudes.

(*a*) A set of 5 double exposures on HSP plates (one pair on each plate) was taken, with exposure times increasing by the factor 4, the first with 14^s each, the last with 1^h each, on a moonless night. The results are given in the curve I, fig. 1. The effect increases within the given limits with increasing total exposure time. A longer total exposure time than 2^h was not suitable because in this case the background would be too dense and make the result uncertain (the aperture of the telescope is 1 : 7). It is remarkable that even on a clear night this sensitive emulsion shows an effect with exposure times of 56^s each. In 14^s no effect is recognizable. (It may be mentioned that the suggestion in the earlier communication, namely, that the effect would appear to be independent of the total exposure

* *M.N.*, **100**, No. 5, 1940.

time, does not hold. The range examined before this paper was written was too small.)

It is possible to obtain a similar result from the experimental curves of Webb and Evans in fig. 4 of their paper, where the densities of pre- and post-fogged exposures are plotted against log exposure in the time-scale and where both intensity- and time-ratio in the mixed exposures are constant, time-ratio being unity. The conditions are thus identical with those of the present experiment. On re-plotting the difference between the post-fog and pre-fog curves against the same abscissæ, the final curve is found to be of similar shape to curve I of fig. 1. The re-plotted curve extends over a considerably greater range of exposure (It) values, and exhibits a maximum.

(*b*) A set of 5 double exposures on HSP plates on a full moon night, with the same time-factor 4, first exposure $3^{s}\cdot5$ each, last 15^{m} each. The results are given in the curve II of fig. 1. Because of the bigger fog intensity the fog effect is bigger than in curve I, it appears earlier and the slope is steeper. A longer exposure time than 15^{m} each was prohibited by the bright background. The effect shows first after an exposure time of 14^{s} each, it is zero at $3^{s}\cdot5$ each.

(*c*) Two sets of 4 double exposures on DXP and BSP plates, on the same full moon night as the previous set (*b*), with the time-factor 4, first exposure $3^{s}\cdot5$ each, last $3^{m}\ 45^{s}$ each. A longer exposure time was omitted because the limits of the Moon's position during the exposure of these 3 sets (hour-angles -1^{h} and $+1^{h}$) were thought to be the greatest which could be allowed to ensure the same brightness of the background. The results are given in the curves III and IV of fig. 1. They differ from one another and from the HSP (curve II). It seems that the fog effect increases with the extension of the emulsion response curve towards the red.

(*d*) Diameter measurement does not show any influence of star magnitudes on fog effect. The deviations from the mean fog effect given by individual stars have nearly the same distributions for bright and faint stars. The mean deviation given by 211 stars on 23 plates, distributed over all measurable magnitudes, was $\pm\ 0^{m}\cdot08$. The results for bright and faint stars separately were the same.

The question of the influence of haze on fog effect, under working conditions, was answered by an exposure of a HSP plate on a moonless night with slight haze, exposure time 15^{m} each. The fog effect was $0^{m}\cdot18$, equal to the fog effect on a clear night (see curve I). Thus, slight haze, without moon, does not increase the fog effect.

As a further illustration of the fog effect a trail plate, taken with the telescope stationary on the NPS, shows that the star trails are nearly always fainter in the later parts. (The very bright stars do not show this effect because, probably owing to Fechner's law, the decrease is not recognizable in the very dense trails.) This phenomenon has afterwards been confirmed on many of the polar trail plates taken on different emulsions at this observatory, of which a considerable stock is available.

6. *The Fog Effect in Spectro-Photometry*

(a) *Influence of the Type of Instrument.*—The magnitude of the effect on spectrographic exposures will clearly depend upon the choice of instrument. It will be greatest for objective-prism cameras of large aperture and short focal length, since the exposure to the diffuse, integrated light from the night-sky background will be effective over the whole of the area of the plate during the trailed exposure to the stellar spectrum. The effect will be much less significant for single-trail spectrograms obtained with slit spectrographs. In such cases, not only is the fogging radiation itself dispersed by the instrument with consequent reduction in the effective intensity at the emulsion surface, but, in addition, the presence of the slit in the optical train will allow only a small proportion of the sky light collected by the objective to pass through the dispersing system.

When the common procedure of trailing the stellar image forwards and backwards across the slit is employed the effects due to each individual trail will be in opposite sense. When integrated they will therefore cancel one another, if it is assumed that the direction in which successive trails are superimposed is without influence on the fog effect for the single trails. Such an assumption is likely to be valid, since the integrated result of multiple exposures using the "Schraffierkassette" method of stellar photometry * is a series of uniformly blackened images.

(b) *Objective-prism Investigations.*—The experimental investigations to be described in this section refer firstly to exposures made with the McClean prismatic camera of twelve inches aperture. Spectra of Polaris were secured with this instrument during the period 1939 February–December on a number of nights, at different lunar phases, and under differing atmospheric conditions. All plates were exposed by trailing the star image in the focal plane of the camera, the telescope being clamped, and stationary. In this way streakiness in the spectrum due to irregularities in the clock-drive was eliminated, and the effective spectral exposure, *i.e.* the time taken for the spectral image to trail a distance equal to its own width (46 μ) was thus maintained constant from one plate to another. In order to evaluate this exposure a plate was exposed for 1, 2, 5, 10 and 15 seconds, shifting the camera in R.A. by a small amount between each exposure. The widths of the recorded spectra were afterwards measured with a micrometer, and a curve connecting image width and exposure time was plotted. The point where the image width began to increase proportionally with exposure time was then noted. From this the effective exposure time was found to be 6 seconds. Since the total exposure time (to sky background) for each plate of the series was 35 minutes, the range of the ratio fogging/spectral exposure from commencement to termination of the mixed exposure were : (*a*) pre-fog, 0–350 ; (*b*) simultaneous fog, 1 ; and (*c*) post-fog, 350–0. Ignoring the effect of simultaneous fog which will be the same at any point in the trailed spectrum, the low intensity fogging exposure will be the sum of the pre- and post-components. The ratio of these components will vary continuously throughout the whole exposure.

* Meyermann and Schwarzschild, *A.N.*, **170**, 278, 1906.

Some spectra were also taken with the 9-inch Kensington prismatic camera in order to compare the fog effect obtained in different instruments; to investigate the behaviour of line intensities; and to test the possible influence of wave-length and density level on the results. The star chosen was γ Cas, and five spectra were taken on one Ilford DXP plate on the night of 1939 November 24 (Moon age 13^d, Z.D. 47°). The exposures were all of 10 minutes' duration, with a single trail to give suitable width and density, the telescope in this case being clock-driven. Owing to the slight longitudinal streakiness from the clock-driven image, each spectrum was treated separately as an integrated unit when measuring in the microphotometer, the length of the slit of the latter being arranged to cover the width of the spectrum (0·4 mm.). The effective exposure, as defined above, for these spectra was 60 seconds, and as this was the exact exposure given in the sensitometer no time-factor entered into the reduction of intensities. The duration of exposure of the latent images to post-fogging light (mean for each spectrum) ranged from 4·5–44·5 minutes, and the ratio fogging/spectral exposure ranged over these same values, except in the case of simultaneous fog for which the value was unity throughout. For these spectra the standardizing spots were impressed on the fogged background of the plate, after exposure in the telescope. These exposure conditions were thus identical with those of the final edge of the last spectrum (all pre-fog, no post-fog), so that intensity measures corresponding to this part of the spectrum represent the true values.

(c) *The Influence of the Intensity of the Sky Background.*—For this investigation with the McClean instrument a series of negatives was obtained on Ilford W 8917 P (panchromatic) emulsion. Each plate was standardized by means of an intensity scale sensitometer in red and blue light, and developed in a standard M.Q. developer * for seven minutes at 65° F. in total darkness. The negatives were afterwards measured in the Oxford microphotometer, all densities being relative to "clear glass." Measures were taken in the spectrum and in the surrounding area fogged by the sky background. Chemical fog was measured as a mean of several values obtained from different parts of each plate, and this value was subsequently subtracted from all other measures of density. The spectral density measures were obtained from the Balmer central absorptions at 6563 and 4340 A., and were made at nine different points across the width of each spectrum, *i.e.* corresponding to post-fogged exposure times of 1·9, 5·8, 9·7, 13·6, 17·5, 21·4, 25·3, 29·2 and 33·1 minutes respectively. The density values so obtained were transformed into intensities relative to the sensitometer scale in the usual manner; and from the known time ratio between sensitometer exposure and exposure in the telescope to the sky background, the intensity of the latter was computed in terms of the same scale. Spectral log intensity values, measured at the two wave-lengths, were then plotted against (post-fogging) exposure time, and it was observed that a straight line could be drawn through the experimental points without undue scatter. Accordingly, the equation to this line was next determined, using the method

* *M.N.*, **100**, 180, 1940.

of least squares, for each wave-length from the results of the complete series of negatives. From the set of linear graphs so obtained, log intensity values corresponding to 0 and 60 minutes post-fogging exposure were evaluated. The difference between these two quantities, expressed on a magnitude scale, represents the mean fog effect per hour.

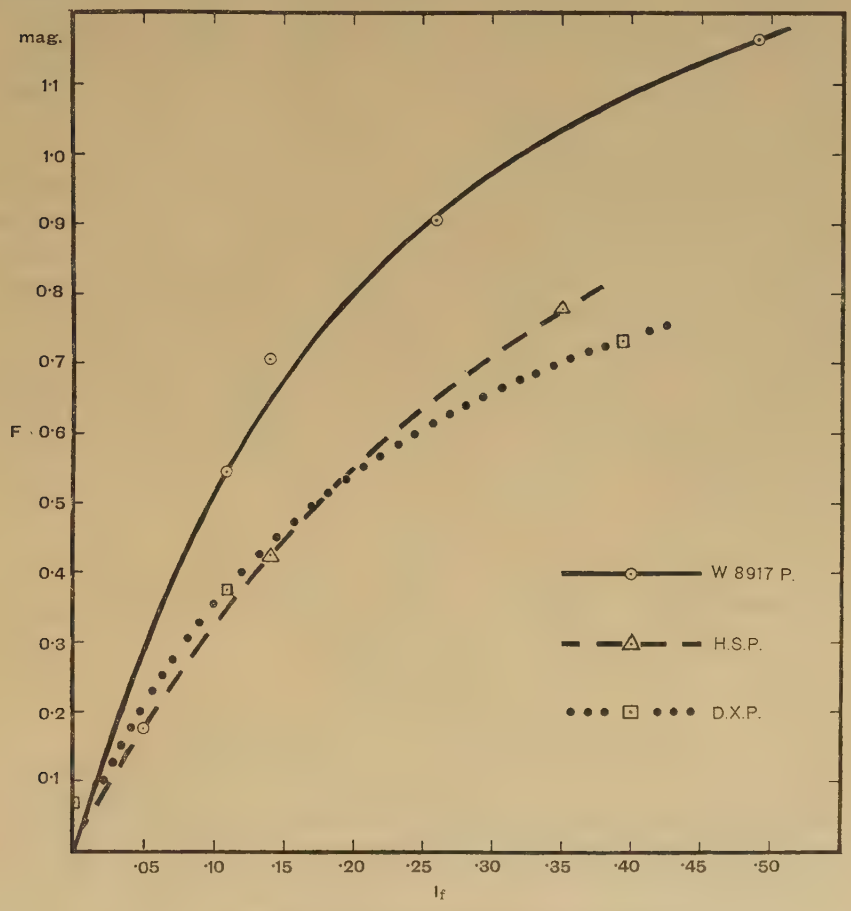


FIG. 2.—*Increase of fog effect with background illumination.*

The series of negatives covers a range of lunar phases from 4 to 16 days, and represents a tenfold increase in the intensity of the diffuse night-sky light. The effective luminosity of the latter will be dependent not only on the intrinsic brilliance and position of the Moon, but will also be influenced by the amount of scatter produced by haze in the atmosphere.

By plotting the fog effect per hour (F) against the observed background intensity (I_f) it is shown in fig. 2 (curve I) that the connexion is not a linear one. For low values of fogging sky light the increase is approximately proportional, but with increasing values of I_f it becomes smaller, thus indicating an approach to a final saturation value of F. The values of F here referred to are the means for the two Balmer lines in each of six spectra

(the point in curve I corresponding to the highest values of F and I_f forms an exception, being based on a single measure at 6563 A.). It may be further mentioned that the variations of the computed true spectral intensities from the mean at each wave-length amount to only 10 per cent. for the complete series. It is concluded, therefore, that the observed variation of the high intensity (spectral) exposure from night to night, due to changes in atmospheric transmission, is not large enough to influence the observed correlation between low intensity (Moon-fog) radiation and the fog effect, F. The observations discussed above cover a range of high/low intensity ratio from 7000 to 700. These results are quite in accordance with those from stellar measurements (section 5 (*b*)).

(d) *The Effect of Density and Spectral Wave-length.*—A partial study of the question whether the fog effect is influenced by the density of the spectrum image, and by the wave-length of the high intensity (spectral) radiation was possible in the γ Cas negatives, where the presence of both emission and absorption in the Balmer lines made it possible to make measures at different density levels in some of these lines. Bromide paper tracings were made with the microphotometer of the contours of four hydrogen lines, and direct readings were taken at the centres of a number of other lines. The following table shows the results obtained from measurements of the tracings, F being the fog effect expressed in magnitude gain per hour of post-fog :—

TABLE I

$H\beta$		$H\gamma$		$H\delta$		$H\zeta$	
D	F	D	F	D	F	D	F
1·15	·30	1·40	·36	1·33	·34	1·12	·32
0·89	·25	1·23	·32	0·98	·30	0·70	·22
0·83	·20	1·00	·42				

These results are insufficient to give a quantitative value for the influence of density, but with one exception (the value of F corresponding to the lowest density value at $H\gamma$) they appear to indicate a slight decrease of F with decreasing density, D, at each wave-length. The exception is based on measurements of the central absorption of $H\gamma$, and is more sensitive to accidental errors from grain clumps than the other two values at this wave-length. The above table gives no indication of a wave-length effect (*i.e.* a change of F with λ at constant values of D) over this range of the spectrum. It is, however, possible for such a change, if present, to be masked by the method used in standardizing the plate. A colour filter was used in the sensitometer, and for wave-lengths near the edge of the transmission band of this filter (centred close to $H\gamma$) the deduced characteristic curve would not be strictly accurate. The fact that both at $H\beta$ and $H\zeta$ the values of F are smaller than at $H\gamma$ lends some support to this view.

The possibility of a wave-length effect was further investigated by examining individual measures obtained from the Polaris spectra at 4340 ($H\gamma$) and 6563 ($H\alpha$). These values of F were first plotted against the fogging intensity, I_f, and smooth curves were drawn through the experi-

mental points. From these curves values of F corresponding to selected values of I_f, at the two wave-lengths considered, were derived, and appear in Table II, cols. 2 and 3. It will be noted that the effect of the post-fogging (night-sky) radiation is greater apparently for blue than for red light in the high intensity (spectral) pre-exposure. Further, this difference, Δ_F (col. 4), is seen to increase as the intensity of the fogging light is augmented.

In the light of the results presented in Table I there is, however, some reason for believing that there is a real dependence of F upon the value of the total image density. If this is confirmed, the fact that the mean total density levels at 6563 A. and 4340 A. were found to be 1·10 and 1·70 respectively indicates that the observed wave-length effect (Table II) is smaller than it would be if there were no difference in density at these two wave-lengths. In the absence of a long series of density measures made at the same wave-length, and covering the entire spectrum, it would seem impossible to separate the two effects completely. A further investigation of the nature of the wave-length effect was made by computing the ratio (ϕ_F) of the fog effect at 4340 A. and 6563 A. for different low intensities, I_f. The results are given in Table II, col. 5.

TABLE II

(1) I_f	(2) F_{4340}	(3) F_{6563}	(4) Δ_F	(5) ϕ_F
0·10	0·52	0·43	0·09	1·21
0·15	0·78	0·62	0·16	1·26
0·20	0·96	0·76	0·20	1·26
0·25	1·05	0·85	0·20	1·23
0·30	1·13	0·93	0·20	1·22
0·35	1·22	0·99	0·23	1·22
0·40	1·30	1·06	0·24	1·22
0·45	1·38	1·12	0·26	1·23

The fact that the ratio, ϕ_F, is sensibly constant, but the difference, Δ_F, variable, over the range of low intensities investigated may have some bearing on the theoretical aspect of the fog effect (see also section 7).

Additional confirmation of the effect of density was sought from the astrographic measures presented in section 5, but these offer no evidence of the dependance of the fog effect on stellar magnitude. It must be remembered, however, that the values of F are derived from measures of star diameters, and it is quite reasonable to suppose that had they been deduced from extra-focal images of the same stars the density effect would have been revealed. A further investigation is clearly indicated on the exact relationship between focal and extra-focal stellar measures (see also section 7).

(e) *Influence of the Emulsion.*—The influence of the intensity of the night-sky light on the fog effect, already described for one particular emulsion, was further examined for a series of eight additional emulsion types. For this purpose a second series of Polaris spectra was taken with the

McClean camera under conditions similar to those appertaining to the former series.

The results of these latter observations were reduced in the manner previously described; but in this instance measures were taken at the Balmer absorptions 4861 ($H\beta$) and 4340 ($H\gamma$) respectively. The emulsions examined comprised blue-sensitive, "orthochromatic" and panchromatic types of different effective speeds. The various brands are listed in Table III. The curves II and III, fig. 2, have been obtained from negatives of this series, and are comparable with curve I, in the same figure. In order to effect a valid comparison of the different emulsions, the measured value of F, for each type was reduced to that which would have been produced by a sky-fog intensity (I_f) level of 0·1.

For the particular instrument used for this survey, the "standard" value of F, as defined above, will approximate closely to that observable on a clear night, with the moon on the meridian, Z.D. 55°, at first quarter.

The results are summarised in Table III, the emulsions being tabulated in order of decreasing response to the post-fogging exposure. The value of F in col. 4 is expressed in magnitudes per hour. Effective emulsion speeds (to blue light) at a density level of 0·5 are given in col. 3 for direct comparison.

It will be seen from the table that, in general, the group emulsion response decreases in the order—panchromatic, "orthochromatic", blue-sensitive, although there are two exceptions which may be significant, viz. Ilford Iso-Zenith and Imperial Eclipse Ortho emulsions. It will be noted also that there is no obvious correlation between the effective speed of the emulsion and the magnitude of the fog effect.

TABLE III

Emulsion	Type	Relative Effective Speed †	F
Ilford Iso-Zenith	Orthochromatic	4·7	0·68 *
,, W 8917 P	Panchromatic	3·5	0·48
,, H.P. 2	,,	6·6	0·39 *
,, H.S.P.	,,	5·2	0·35
,, D.X.P.	Orthochromatic	4·9	0·32
Barnet S.P.	Blue-sensitive	3·9	0·27
Ilford Zenith	,,	3·9	0·20 *
Imp. Eclipse O.	Orthochromatic	4·3	0·00 *
Ilford Ordinary	Blue-sensitive	1·0	0·00 *

* Denotes value of F was obtained from a single negative.

† The effective speed is the quantity $(10^4/E)$ where E is the sensitometer exposure (I_t) required to produce a density 0·5.

It should be mentioned here that the values obtained for the fog effect, using different emulsions in the case of the astrographic exposures (see section 5 *c*, and fig. 1), conform to those given in Table III.

(f) *Influence of the Instrumental Aperture.*—In order to investigate the influence of the optical qualities of the instrument on the fog effect, the

latter was determined from direct readings of $H\gamma$ (central absorption) in the γ Cas negatives taken with the 9-inch prismatic camera, and compared with the values obtained at the same wave-length in Polaris from negatives of the 12-inch instrument. The emulsions used, as well as the methods of standardizing, measurement and reduction, were identical in the two cases, while the atmospheric conditions and moonlight were similar. The resulting values of F (0·87 and 0·44 for the 12-inch and 9-inch instruments respectively) are nearly proportional to the squares of the respective apertures. This result might be expected from the relation between F and fogging intensity described above, but in addition to serving as a confirmation of this relation it indicates that no instrumental factor, other than the aperture and rate of trail, has a significant bearing on the fog effect.

(g) *The Effect of Post-fogging on Line Contours.*—There are two ways in which the fog effect may produce errors in the observed contour of a line. In the first place all intensities will be increased, relatively to the plate background, and secondly (if there is a density effect), different portions of the contour will be affected differently and therefore displaced relatively to each other. In other words, the contour will be both raised (in relation to the plate background as zero) and elongated. In computing intensities—either central percentage absorptions or total absorptions—these two influences will tend to operate in opposite directions. For example, the area enclosed by the contour would be increased (through the elongation at constant width) thus tending to increase the measured total absorption, but the increased distance of the base of the line from the zero-level would tend to decrease the total absorption (measured as an equivalent width). A theoretical determination of the resultant effect would be of little value without more accurate numerical data, but it is evident that these two opposing influences may not cancel out, and that there may be a residual effect on the line intensity. To investigate this possibility experimentally careful measures were made of the absorption contours of $H\gamma$, $H\delta$ and $H\zeta$ in the γ Cas spectra. The complete contours were not measured, owing to the presence of the emission-line contours on either side of the absorption. That part of the absorption below its intersection by the estimated spectrum background was measured for each of the three lines, while for $H\gamma$ and $H\delta$ the part below a line drawn through the two maxima of the emission components was also measured. Neither method gives the true values of the total absorptions, but this is immaterial for the present purpose, which is to investigate variations only.

The results are given in Table IV. Total absorptions are expressed in the form $10^6 \cdot w/\lambda$, where w is the equivalent width in angstroms ; a and b refer to absorptions from the spectrum background and the level of emission maxima respectively ; central intensities are percentages of complete absorption from the spectrum background.

Apart from accidental errors of measurement, the total absorptions show a definite decrease in value proportional to the time of post-fogging exposure. Their true values may be obtained by extrapolating the straight line defining this relation back to zero post-fog, and these computed values

are entered in the table. The value of F (the decrease in the measured total absorption per hour of post-fog) was also computed from this relation, and is given in the last line. It will be noticed that F is not constant but, with the exception of Hζ, is roughly proportional to the actual value of the total absorption itself. In the case of Hζ errors may have been introduced as mentioned above, on account of its wave-length being near the edge of the transmission band of the sensitometer filter.

TABLE IV

Variations of Line Intensities with Post-fogging Exposure

Post-fogging Time	Total Absorptions					Central Intensities		
	Hγ		*Hδ*		*Hζ*			
	a	*b*	*a*	*b*	*a*	*Hγ*	*Hδ*	*Hζ*
44·5	99	212	98	158	226	47	58	85
34·5	106	221	102	173	239	51	58	84
24·5	108	224	107	180	234	52	55	83
14·5	106	230	112	180	260	54	58	83
4·5	113	242	122	188	240	53	59	85
(0·0)	114	244	122	193	258			
F	20	44	34	46	36			

The central intensities show no such definite effect of post-fog. If the mean of the last three columns is taken there is an indication of a decrease of about 3 per cent. per hour, but this is of the same order of magnitude as accidental errors of measurement, and it would seem that in most cases it would be safe to ignore the fog effect in measurements of central intensity —especially if the moonlight is not strong, or if a slit spectrograph is used.

7. *Theoretical Treatment*

The foregoing experimental results appear to be in general qualitative agreement with the theory of Gurney and Mott * †, and with the later work of Webb and Evans.‡ These theories, therefore, supply a reasonable basis for the explanation of the observed effects. The theory as it stands, however, does not suggest a dependance of the fog effect on the wave-length of the high-intensity (star or spectrum) exciting radiation, though the present results (section 6 (d)) indicate the possible existence of such a dependance. No complete interpretation of this can be attempted without a systematic investigation of the effect throughout the spectrum. The observed difference, Δ_F, and the constancy of the ratio ϕ_F (see Table II), may, however, arise from the fact (found experimentally for this emulsion) that reciprocity failure at low intensities decreases with increasing wave-length, and from the different sensitivities of the emulsion at the two wave-lengths.

* *Proc. Roy. Soc.*, A, **164**, 151, 1938. † Mott, *Phot. J.*, **78**, 286, 1938.
‡ *Comms. Nos.* 674, 687 from Kodak Research Labs., 1938.

The "density effect" mentioned in section 6 (d) is also not readily explained, and the following laboratory experiments were made to investigate it more fully. A range of high-intensity light sources was obtained by using a tungsten-filament lamp and diffusing screen behind a neutral wedge, and the same source, diminished in intensity by suitable silver-absorbing screens, was used as fogging light. All exposures were of 10 seconds' duration.

Three wedge images were photographed on each negative, the order being: (1) wedge with post-fog, (2) wedge with no fog, (3) wedge with pre-fog. The plate was suitably masked so that the second exposure was unaffected by the fogging exposure which was applied simultaneously in the case of (1) and (3). It was also arranged that the fogged areas should extend beyond the wedge images in order that the density (and intensity)

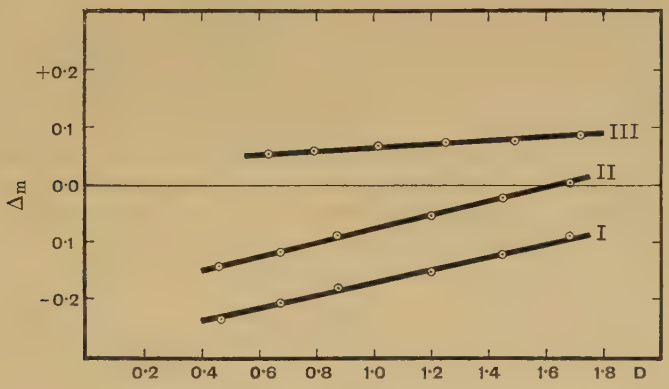


FIG. 3.—*Changes in the effect of pre-fog (I), post-fog (II), and combination of both (III), with density.*

of the fog background could be measured independently. Two panchromatic plates were exposed to different intensities of fogging light, using the same high-intensity exposure. The plates were calibrated in the usual way with the tube sensitometer, the exposure time being the same as for the main exposures. All densities were measured against clear glass in the microphotometer. Using the spot calibration curve, intensities in the wedge images corresponding to six different transmission levels were computed, together with the intensity of the fogging exposure. The intensities computed from the densities of the post- and pre-fogged wedge images were next corrected by simple subtraction of the fog intensity, and these corrected values were finally compared with those from the central wedge image which had received no fog. Differences were plotted on a magnitude scale against the measured image density. Three curves are reproduced in fig. 3 from one of the negatives only, I and II representing pre-fogged minus normal wedge and post-fogged minus normal wedge, respectively. Curve III illustrates the fog effect, as defined earlier in this paper—*i.e.* the difference between post- and pre-fogged wedge images. It will be noticed that over the range of measurements there is a linear relation between the log intensity difference and the density, and that all three lines have positive gradients.

The former result confirms the existence of the small density effect discussed in a previous section. The curves obtained from the second negative, at a higher level of background fog (not reproduced), show the general relationship between fog effect and density to be similar, but their gradients have increased, indicating an influence of the low-intensity component of the mixed exposure upon the density effect. It would appear somewhat difficult, on theoretical grounds, to account for the existence of an effect which is a function of the total density only, and it therefore seems likely that the "density effect" is determined by the combined influence of high- and low-intensity components of the mixed exposure. It is also apparent, from a study of the curves, that both pre- and post-fog have an influence on the final density of the image, although this may only be the case for low values of the high/low intensity ratio, as in this experiment. For much higher values of this ratio (resulting from greatly increased values of the high-intensity component) the theory would suggest that the post-fog curve would tend to coincide with the no fog (zero) curve, and would do so ultimately if all reciprocity failure is eliminated.

On the basis of the above-mentioned theory a tentative suggestion is made in order to explain the results of the wedge experiments presented in fig. 3. The effect of a high intensity pre-exposure, whose value is below the optimal level, will be twofold. Considering first those nuclei which had, *ab initio*, a few silver atoms attached to them, it may be supposed that all of these become stable latent image specks. Of those which did not possess any *Ag* atoms originally, some will be stabilised, and a certain number of the remainder will acquire identical silver concentrations below that required to produce stabilisation. One effect of increasing the high intensity exposure will be to increase the number of partially stabilised specks, and a proportion of these will be finally stabilised by the subsequent low intensity exposure. Further, at each high intensity level in the wedge image, the effect of increasing the low intensity post-exposure will be to increase the fraction of partially prepared specks which are finally rendered developable. The effect of a low intensity pre-exposure will only be to stabilise a proportion of the specks containing silver, and the final density resulting from the mixed exposure will, in consequence, be less. An explanation is thus offered of the observed "density effect"—viz. the positive gradient of each curve in fig. 3, and the increase of their gradients which results from an augmented low intensity (fogging) exposure.

Acknowledgment is due to Mr. E. R. Davies, Director of the Kodak Research Laboratory, Harrow, for his further kindness in lending a selection of literature dealing with the effects of auxiliary exposure.

Summary

The magnitude or density difference between two successive equal exposures on one plate is shown to be due almost entirely to the effect of fogging background light.

A short historical survey is given, and it is shown that the humidity effect suggested by early workers is too small to be of much significance.

The magnitude of the fog effect for different instruments is determined both for star diameters and spectral images.

It is further shown that the photographic emulsion, the intensity of the night sky, the wave-length of the light in the principal exposures, the final density of the image, the exposure times and the aperture of the instrument all have an influence on the magnitude of the effect.

A theoretical explanation of the observed facts is suggested, based on a modern latent image theory.

Definite corrections arising from this effect cannot be given, but must be determined by individual observers to meet the conditions of their own work.

The Norman Lockyer Observatory,
Sidmouth :
1940 *March* 1.

ON THE ORIGIN OF THE SOLAR SYSTEM

R. A. Lyttleton, M.A., Ph.D.

Introduction.—The present paper has been written with a view to clarifying the position of the problem of the origin of the solar system. As will be recalled, a few years ago a new initial hypothesis for the formulation of a theory of the origin of the planets was put forward by the writer in order to satisfy the requirements of angular momentum without necessarily sacrificing the details of the available theory propounded by Jeans and Jeffreys. Since this time the whole of the theory has been subjected to a considerable amount of discussion, a great deal of which has been of a critical kind. In view of the fact that much of this criticism has arisen through a misunderstanding of the nature of the problem, it will not be out of place to consider to begin with exactly what is the problem that has to be solved, and, moreover, what methods are to be adopted in obtaining a suitable theory.

1. *The Nature of the Problem.*—In the first place, it may be noticed that while cosmogony is often regarded as a speculative subject, it is nevertheless important to realise that its problems deserve full consideration, since they may prove equally as fruitful as the detailed investigations of those problems that can be more closely defined. In addition to this, one of the tasks of theoretical astronomy is to attempt to bring all observable phenomena within a consistent scheme depending on as few rules as possible, and therefore such problems as the present one are perfectly legitimate topics for scientific investigation.

In the problem of the origin of the planets it is clear that the chief rules to be satisfied are the laws of dynamics, and attention must in the first place be directed to them. An immediate difficulty is here met with, however, for many bodies are involved in the discussion, and these move at times at mutual distances from each other that preclude all hope of accurate use of the equations of motion. Furthermore, it is an essential part of the theory that at some time during the motion a collision takes place between two of the stars, and it is apparent that the effects of this collision cannot be decided accurately. Nevertheless, by taking advantage of certain features of the motion and making simplifying approximations, it proves possible to obtain satisfactory indications of the paths of the bodies involved. In principle this is equivalent to the standard practice of celestial mechanics in which parts of the disturbing forces have inevitably to be disregarded. It is therefore clear that for the case under discussion it would be altogether unjustifiable to attempt to calculate the motion in great detail. It would be unscientific to make calculations having a pretence to accuracy when the assumptions on which they depend cannot in the nature of the problem be stated in precise terms. The most that can reasonably be sought for is a theory that satisfies the broader dynamical conditions laid down by energy and momentum considerations. Particularly is this the case in the early stages of a theory of

the present kind; indeed such considerations have in the past sufficed to eliminate many proposed hypotheses for the origin of the solar system.

A second point worth mentioning concerns the question of the choice of masses and initial velocities of the bodies involved. Except that a particular one of the bodies must be identified with the Sun, the remaining quantities are arbitrary. It must be remembered, however, that the choice of definite values for these masses and velocities would not constitute *ad hoc* assumptions in the theory. It is the differential nature of the laws of dynamics that enables them to resume such a wide range of phenomena. The constants that appear on integrating them, in general can only be determined by observation of the system under discussion, for it has to be assumed that the universe can be subdivided into complete systems. For example, the masses of the planets and the eccentricities of their orbits are not *ad hoc* assumptions of planetary theory. In developing such theory, at certain stages of the procedure, arbitrary constants appear to which definite numerical values have to be assigned; their values cannot be determined by theoretical means. This is all so well known and generally accepted by long-established custom, that it appears to have been lost sight of that in the problem of the origin of the planets there must be exactly analogous constants to which definite values must also be assigned. Even if it were found that these latter constants were essentially restricted to the narrowest ranges, as happens in planetary theory, this could not be urged as an argument against the theory. To say that such a feature renders the theory highly improbable would be as absurd as to urge the same objection to the planetary theory. If this point of view appears invalid to the reader it should not be dismissed on account of its seeming novelty, for it is through the misunderstanding of this very question that the majority of objections to the present theory of the origin of the planets have their source. Although the point is philosophically a simple one, it may be worth while to illustrate it further by supposing for the moment that two dynamically consistent theories were available. To attempt to decide between them by considerations of the type used in probability arguments would clearly be impossible. If, however, the first of the two theories could be satisfied by a much wider range of the constants than in the second theory, then it could only be decided that planetary systems are much more likely to be formed by the first process than by the second.

2. *The Amount of Material removed from the Parent Stars.*—Consider now the solution of the problem of the origin of the planets proposed by the writer. The double-star hypothesis was suggested as a modification of the Jeans and Jeffreys theory in order to resolve the angular momentum difficulty noticed by Russell. In the discussion of this hypothesis given in my first paper * no account was taken of the energy required to remove the planets from the surfaces of the stars. This omission, which was pointed out by Hill and Luyten †, was simply a survival from the tidal theory of Jeans and Jeffreys. It is instructive to consider this point in the light of the foregoing remarks. Any attempt to test the double-star hypothesis by a detailed procedure, as for instance by calculating the paths by numerical integrations,

* *M.N.*, **96**, 559, 1936. † *Ap. J.*, **86**, 470, 1937.

as some writers have suggested, would have been valueless compared with the test provided by the new feature of the motion brought to light by Hill and Luyten. Moreover, in support of the method of testing any hypothesis by reference to broad principles, it may be mentioned that this method has been adopted by all previous investigators.

In the Jeans and Jeffreys form of the theory the encounter between the Sun and the star had to be of the so-called intermediate type, and in these circumstances the relative velocity would not be large enough for the stars to escape from each other and at the same time remove sufficient planetary material. That this objection applies to the original tidal theory has not previously been noticed as far as the present writer is aware. This point seems sufficient in itself to indicate that the encounter must take the form of a collision.

As far as the double-star hypothesis is concerned the additional energy required does not prove in any way fatal, but merely restricts the form of the encounter somewhat. The energy has to be provided by the intruding star, and the amount available can be increased sufficiently, without at the same time preventing the planets from being captured by the Sun, by increasing the velocity of this star. The necessary modifications have already been considered.* There is, however, one point in the theory that perhaps should be further emphasised, as Professor Luyten appears to have overlooked it in a recent article.† Briefly, the theory envisages the primitive planets as having been formed by condensations in the filament that stretches out between the two parent stars after their collision. The actual capture of these planets by the Sun could not take place until several years after the collision, when the parent stars would have had time to recede to some considerable distance. At a very early stage the planets would already have condensed into comparatively small masses, indeed the filament might have condensed into a single mass. Now it has been maintained by Luyten that if such a planet is to escape from the parent stars it must lie in a certain region occupying no more than a few per cent. of the region intermediate between the two stars. The linear extent of such a region would nevertheless be of the order of a few astronomical units, and the theory supposes that the primitive planets initially occupied this region. In my earlier reply to Luyten's criticism I have actually shown, with simplifying conditions, that the planets would tend to occupy the required position. This is the part of the theory that Luyten seems to have overlooked. It is necessary only that planets occupy the 6 per cent. of the region (to use Luyten's figure), and it would be wrong to insist on filling up the remaining 94 per cent. with planetary material endowed with escape velocity ; there would be no justification for such a step. In view of the smallness of the planets they would in fact occupy only a minute fraction of an astronomical unit, the space between them being devoid of planets. The approximation of a linear distribution of velocities with position does not imply or require a uniform distribution of matter, even if the initial form of the filament were approximately uniform. The various portions would condense by internal forces for the most part,

* *M.N.*, 98, 536, 1938. † *M.N.*, 99, 692, 1939.

and the velocities of their centres of mass would roughly correspond to their positions and therefore remain linearly distributed. Owing to the fact that Luyten has inadvertently ignored this feature of the problem, a superfluous factor of 16 is introduced and the amount of energy then required is proportionately increased. Nevertheless, the writer went so far as to show that a reasonable encounter could be devised in which even this needlessly high amount of energy could be supplied.

The foregoing point concerning the region in which the planets must move seemed sufficiently obvious without detailed explanation at the time of writing my paper (1938), and was dealt with as follows in it: "The region in which a planet must move if it is to escape from both stars may be only a small fraction of the space between them, but it is not essential to suppose that the filament would aggregate into a large number of planets ranging right from one star to the other." *

3. *The Compositions of the Major Planets.*—In discussing the condensation of the planets from a filament at high temperature, Luyten has pointed out that such an element as hydrogen would readily escape. The presence of hydrogen on the major planets is therefore regarded by Luyten as proof that these bodies can never have been at high temperature. This argument is very obviously fallacious, however. It seems almost certain that the lightest elements would in the first place escape from the filament or primitive planet, but such gases would not necessarily escape from the Sun and, possessed of angular momentum, would form a tenuous medium through which the planets must move unless their orbits became circular. The details have been discussed by Jeffreys. † It would be sufficient for these planets to form initially from the heavier elements, and it is entirely possible that these primitive planets could pick up the lighter gases subsequently when their surfaces were at suitable temperatures. ‡ This suggestion was also mentioned in my earlier paper, p. 542, at the end of section 4. It is of interest to notice that this description of the origin of the atmospheres of the great planets accords well with the fissional theory of their development. For it would mean that the primitive rotating planets could be regarded as formed mainly of material of high weight such as silicon, iron, etc., and accordingly would be in liquid form and therefore undergo disruption by fission; while the subsequent addition of extensive atmospheres would not only lower the mean densities but would lengthen their rotation periods, a requirement that appears to be necessary from a discussion of the fission question. §

4. *The Energy required to Form the Planets.*—Bearing in mind the discussions of the last two sections, the amount of energy required to remove the planetary material can now be calculated. In the first place, it is clear that the whole amount of material captured by the Sun need not greatly exceed the present mass of the planets and may be taken as, say, 0·002○. In addition to this some estimate has to be made of the parts of the filament reabsorbed by the parent stars, and in the absence of definite information on the point it may reasonably be taken as twice the previous amount, viz.

* *Loc. cit.*, p 543.
† *The Earth*, chap. iv.
‡ *Proc. Camb. Phil. Soc.*, **35**, 592, 1939.
§ *M.N.*, **98**, 643, 1938.

0·004⊙. This material, it is to be noticed, is not endowed with the escape velocity at the surface of the stars and its energy will depend on its initial distribution. As far as velocity is concerned, the most reasonable supposition would appear to be the simple one of a linear distribution with position between the velocities of the two stars. The actual distribution of the material is more difficult to assess. In my earlier paper it was taken to be the same at all distances from the star. On this basis the energy given to this material is readily found to be one-third of the energy that could cause it to escape from the star. It seems more probable that a greater portion of the material would receive small velocity than high velocity, and if the distribution is taken as diminishing uniformly with height above the stars, then it is found that the material possesses one-twelfth the energy that could cause it to escape. Accordingly it will not be unreasonable if the total energy required is made up as follows :—

> Mass of planets 0·002⊙ to be given escape energy,
> together with a mass of 0·004⊙ to be given 1/3 the escape energy.

Thus the intruding star has to provide

$$\tfrac{1}{2} \times 0{\cdot}0033 \times 620^2 = 640 \text{ units of energy.}$$

This energy required to remove the planetary material is supplied by the colliding stars, and must come from the energy possessed by them just before the collision. If M_1 denotes the Sun's companion, M_2 the intruder, and U_1 and U_2 the relative velocities of these two stars at such times before and after the collision when their distances apart are the same, the loss of energy is

$$\frac{1}{2} \cdot \frac{M_1 M_2}{M_1 + M_2} \cdot (U_1^2 - U_2^2).$$

It is of some interest to consider the mass factor $\frac{1}{2} \cdot \frac{M_1 M_2}{M_1 + M_2}$, as rather too much significance has been attached to the particular values chosen for the masses in my earlier paper. If $M_1 = 2\odot$ and $M_2 = 8\odot$ this factor is 4/5, while if $M_1 = 2\odot$ and $M_2 = 4\odot$ this factor is 2/3, and so on. It is immediately clear that the amount of energy is not highly sensitive to changes in M_2, and that a reduction of M_2 by one-half from 8 to 4 scarcely affects the energy appreciably. In my previous paper the motion of the intermediate material was considered in the case $M_2 = 4M_1$, and the values $M_1 = 2\odot$ and $M_2 = 8\odot$ were chosen to agree with this calculation rather than for any particular merit belonging to stars of mass 8⊙.

It is necessary, therefore, that

$$\frac{1}{2} \cdot \frac{M_1 M_2}{M_1 + M_2} \cdot (U_1^2 - U_2^2) > 640$$

or

$$U_1^2 - U_2^2 > 960 (< 35^2 - 15^2).$$

Hence the requirements of the collision can be satisfied by considerably smaller values of the velocities than given in my earlier paper. In this former paper the writer devised a collision that could supply 1000 units of energy, in the belief that if this were done it would be regarded as sufficient

argument that a suitable collision could also be devised, though more readily in the case of the smaller amount that is actually necessary. Even if $M_1 = M_2 = 2\odot$, the necessary energy could be supplied if

$$U_1^2 - U_2^2 > 1280 < 40^2 - 15^2.$$

These velocities seem in no way unreasonable for stars of mass similar to those involved. For stars of mass of the order of that of the Sun would be expected to have a peculiar velocity of the order of 20 km./sec., and a relative velocity of the above order does not appear to be unacceptably high. From these remarks it is seen that it would not be right to maintain that the energy could only be supplied by a star of exceptionally high mass.

5. *Other Criticisms.*—A number of other criticisms have been urged against the theory from time to time that may properly be considered briefly here. In the first place, it may again be mentioned that in confining attention to the broader dynamical principles in discussing the motion, the present theory follows the method adopted by all previous investigators. That this method can be justified may easily be seen, for it has proved sufficient to dispose of all previous hypotheses, including the Laplacian hypothesis, the Planetesimal hypothesis of Chamberlin and Moulton, and the tidal and collision theories of Jeans and Jeffreys. Moreover, the method can, in the present case, be supported by a far more cogent reason than this, for the nature of the problem is such that it cannot be sufficiently closely defined to warrant excessively detailed calculations. It would be to commit an error of method if such calculations were undertaken in the early stages of any theory, while in the present instance it is doubtful if the initial values of the constants could ever be given sufficiently precisely to make any detailed calculation possible. The whole history of this problem has shown how slow progress can be, and consideration of the errors that have already come to light shows readily enough that detailed analysis has not been necessary. The present hypothesis has been under discussion now for only four years—quite a short interval compared with the periods between progressive steps in this problem.

It has further been objected that the present theory is not sufficiently simple to be regarded as providing a satisfactory solution of the problem. Here again it can only be answered that such a criticism is not a valid objection, the complexity of the motion is not an argument against the theory. A simple solution of a problem does not exist in the scientific sense until one has been invented, and in the case of the planetary system there seems to be no *a priori* reason why any simpler explanation of its origin than the present one should exist. Indeed a three-body encounter seems unavoidable in view of the fact established by Russell that the planets could not have been separated from the Sun with adequate angular momentum. It is entirely possible that the planetary configuration is, in its broader features of small eccentricities and inclinations, the only dynamically stable arrangement for a number of small bodies moving near a massive body. If this is so, the present orbits could not be used as a guide to the mode of origin of the system. This of course was Laplace's unwitting error.

In his earlier criticisms Luyten regarded the present theory as untenable on the grounds that the decisive moment for capture is when the parent stars have only just separated after the collision and the filament is still uncondensed.* It is quite evident, however, that the attraction of the Sun at this time would be negligible compared with that of the parent stars. In his later paper Luyten has apparently conceded this point †, without explicit acknowledgment, but now maintains that the Sun and planets would be very unlikely to move in similar directions for any protracted period of time. In view of the fact that my diagram of the motions of the bodies shows that the planets and the Sun attain similar velocities and, after the collision, are being acted on by similar forces, their similarities of motion appear to be wholly reasonable consequences. For example, at certain times the planets Neptune and Uranus move along roughly parallel paths for many years.

Luyten further considers that there is no evidence that the solar system could have been generated by external action.‡ Sufficient answer to this would appear to be contained in the fact that the system is such that over 98 per cent. of the angular momentum is possessed by less than 1/5 per cent. of the mass of the system. This has seemed to most writers on the subject during the present century to be overwhelming evidence in favour of invoking some agency apart from the Sun.

6. *The Assumptions of the Theory.*—Finally may be considered the so-called assumptions of the theory. It is an essential part of the hypothesis that a collision takes place, but this is not an assumption. In the very nature of the problem definite values *must* be selected for the initial velocities, etc., and that a choice of these can be made in such a way that a collision takes place, is exactly the advantage of the hypothesis when coupled with the fact that the angular momentum difficulty is simultaneously resolved.

The first assumption made in the theory relates to the filament. As the generally accepted views concerning the development of the filament were current in cosmogony long before the double-star hypothesis was put forward, the writer does not feel complete responsibility for them. On the other hand, the remarks of section 3 of this paper go a considerable way towards resolving the difficulty of the condensing of the filament, as also does to some extent the fissional theory of the common origin of Jupiter and Saturn. This difficulty, of course, has been present in cosmogony from the earliest times, even Laplace's hypothesis containing it in some degree. As the sole available source of matter in the right form (*i.e.* with all the heavier elements present) is the stars, the only immediate prospect of reducing the difficulty still further would seem to lie in assigning the origin of the planets to stars of as low internal temperature as possible. In considering this suggestion it is perhaps worth while to bear in mind that our knowledge of the internal temperature distributions of such stars cannot yet be regarded as definitive.

The second assumption concerns the location of the planets relative to the parent stars. The present theory supposes that they would lie in the

* *Observatory*, March 1938, p. 84.
† *M.N.*, **99**, 694, 1939.
‡ *Observatory*, March 1938, p. 84.

region suitable for capture by the Sun, maintained by Luyten to be 6 per cent. of the intervening region. It is difficult to see any objection to this assumption. The mode of formation of the planets is such that the calculation of their motion under the influence of two strong gravitational centres could only be made by introducing very special hypotheses to describe the mechanism of this formation. At the present time there seems to be no available question of a similar nature that could be used as a guide, so that no reliance could be placed on any dependent calculations. If for simplicity the filament is regarded as condensing into a single primitive planet, it is clear that in the case of equal masses of the parent stars this body would lie in the region suitable for capture by the Sun. For a mass ratio of 4 : 1 the suitable region is near the point of trisection of M_1M_2 that is nearer the less massive star. Furthermore, the 6 per cent. referred to by Luyten is calculated on the assumption that M_1 and M_2 are separating with parabolic velocity. This assumption clearly leads to the lower limit for the estimate of the suitable region. This may readily be seen by supposing the mass M_2 to be negligibly small and considering a filament with linearly distributed velocity. At some point of its length the velocity will equal the parabolic velocity and all material beyond this will escape. No limit need be set to the linear extent of this material, and it is clear that the neglect of its mass does not alter the conclusion, apart from trivial considerations. Thus it appears that what available evidence there is justifies this second assumption of the theory.

Conclusion.—The discussion may perhaps be concluded with some general remarks on the theory of the origin of the solar system. Owing to the fact that we are aware of only one solar system, the solution of the problem does not have to satisfy any requirements implying the frequent occurrence of planetary systems in space. From the point of view of scientific method the problem of the origin of binary stars, for example, is on a different footing from the present problem. As a matter of strict logic, however, the two problems need not be considered as demanding different types of solution. It would be possible to explain each binary system by means of a special hypothesis, but experience has shown that in general for problems of this kind a single explanation will suffice, and only such a solution of the problem of the origin of binary stars would be regarded as satisfactory. Nevertheless, given such a solution, it would not be possible with reference to an individual selected binary system to decide that it had necessarily originated in accordance with the solution. In just the same way if it were known that a large number of planetary systems exist, it would be reasonable to expect that they could all be explained in terms of a single theory.

It may yet prove possible to find an explanation of our own system that makes it highly probable that other stars also possess planets, but if such an explanation is eventually forthcoming it can never be known with certainty whether our system originated in the manner the theory suggests. Accordingly it may prove possible to find several explanations of our system. The fact, therefore, that the theory discussed in the present paper is not open to any objection need not therefore be regarded as in any way precluding the possibility that other explanations may be invented.

THE SOLAR CORONA OF 1937 JUNE 8*

I. L. Thomsen

Observation Spot.—From the time of landing on Canton Island, to the morning of the eclipse, eleven days were available. Determination of the meridian, general work and nights spoiled by passing clouds left only three nights for latitude and longitude observations. A 5-inch Cooke vernier theodolite reading to 20″ was used. One division of the vernier level equalled 28″·2. The result of the observations gave:

Latitude $-2^\circ\ 49'\ 08''.4$

Longitude $\left.\begin{matrix} 11^h\ 26^m\ 51^s.32 \\ 171^\circ\ 42'\ 50'' \end{matrix}\right\}$ West

$\log \rho \sin \phi' = 8.68887\ n$

$\log \rho \cos \phi' = 9.99948$

Height: A few feet above sea-level.

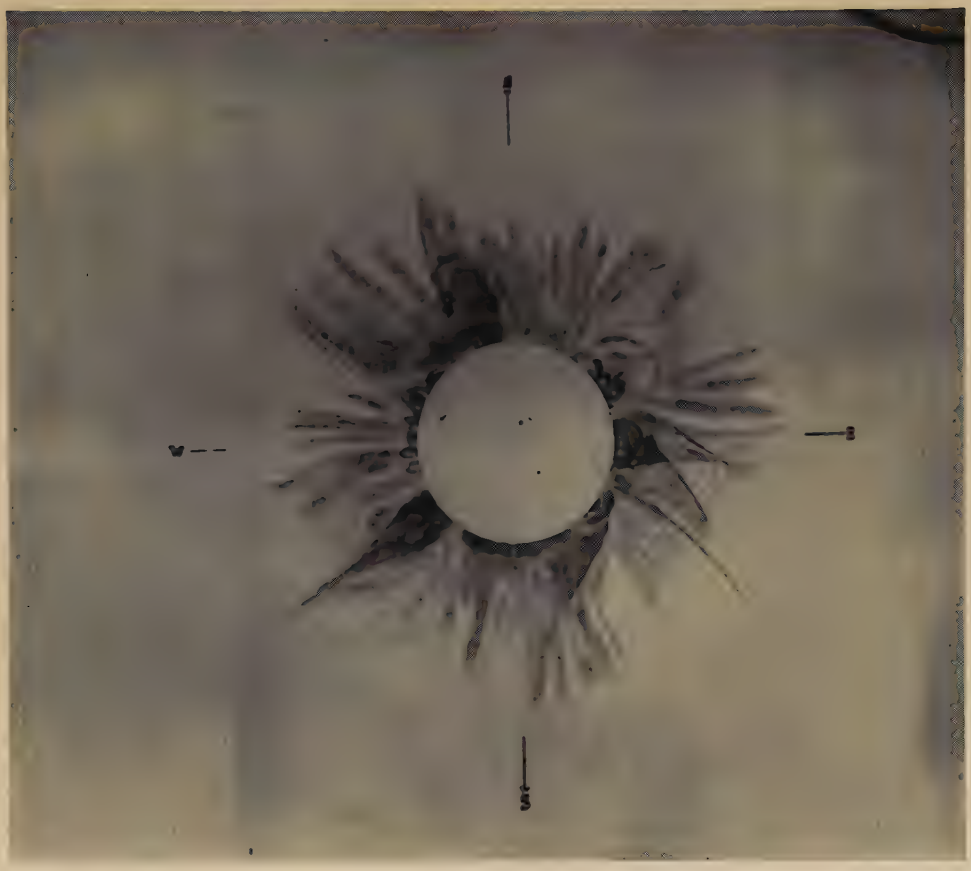
Time was kept by means of a chronometer checked by short-wave radio time-signals from NAA, NPM, NPG and FZR, and was considered to be reliable to 0·1 second.

Contact Times.—Although it was not considered an object of the expedition, an attempt was made to time the contacts. This was done by three observers as follows: (1) by the Navigating Officer of H.M.S. *Wellington*, Lieut. Butler-Bowden, R.N., using the theodolite telescope and a stop-watch; (2) by Mr. M. Geddes observing the flash spectrum with prism binoculars and stop-watch; (3) by the writer on the image on the focussing screen. The resulting times, however, cannot compare with the method of chords recorded photographically, and are given only for record purposes. The computed times were derived from the *Nautical Almanac* elements and the observed position.

Phenomenon	Calculated Time (U.T.) 1937 June 8	Observed Time (U.T.)		
		(1)	(2)	(3)
	h m s	s	s	s
First contact	18 5 37·6	59·0		48·8
Second contact	19 6 9·2	23·3	15·2	
Mid-eclipse	19 7 55·5	69·6	65·3	
Third contact	19 9 41·8	55·8	55·4	
Fourth contact	20 19 46·7	47·1		41·2
Duration of totality	3 32·6	32·5	40·2	

It is felt that the times of (1) are the best. This observer also acted as time-keeper, and the coronagraph shutter was first opened at the word "Go",

* See also C. B. Michie, *M.N.*, **99**, 132, 1938.



Drawing of 1937 corona as deduced from the negatives.

and corresponds to his time for second contact. The photograph shows that this might have been very slightly early. For the last photograph, the shutter was held open until the word "Stop". The resulting photograph shows that this must have definitely been late, although in this case (1) and (2) agree very well.

Coronagraph.—The 19-foot focal length 4-inch aperture coronagraph of the Royal Irish Academy was used, and its erection and adjustment left no time for trial focussing or orientation of plates. The former was done visually and the latter by using the calculated position angle of third contact which was recorded on the last photograph. Comparison of the position angles determined on the corona photographs with a solar prominence plate (June 8^{d} 20^{h} 3^{m} U.T.), kindly lent by Dr. Nicholson of Mt. Wilson Observatory, showed that there is a systematic error of $+5°$ in the sense Canton Island minus Mt. Wilson.

During totality, nine exposures of 2, 5, 10, 20, 45, 25, 20, 2 and 10 seconds duration were made on Ilford Panchromatic Plates, H. & D. 400. The shortest exposures show the corona to nearly 0·2 solar diameter, and the longest to nearly one diameter with suspicions of further faint extensions.

General Appearance of Corona.—The corona appeared to be symmetrical, but excentric with respect to the centre of the Sun. The north-east quadrant was the most abundant in coronal material. Visual observers and cameras with small *f*-ratios recorded streamers and coronal details to much greater distances than the 19-foot coronagraph.

Detailed Coronal Structure.—A drawing was made of the corona (Plate 2) after examining all the negatives as carefully as possible. The cardinal points and all position angles are referred to the north pole of the Sun, the systematic error mentioned above being kept in mind.

Precise description of all coronal details and structures is possibly just as difficult as it ever has been in the past. The existence of a corona in the form of an undisturbed atmosphere surrounding the Sun, diminishing in density with increasing height, is possibly the first approximation in our ideas. Superimposed on this general atmosphere by some solar force are the great coronal features which are so well known. From the work done in drawing the corona and considering the negatives for a considerable time, the conviction has gradually grown that if such a more or less general coronal atmosphere exists, it is so rarified and faint as to be almost impossible of observation by ordinary methods. Every detail and darkening in the correctly exposed coronal regions on the negatives is believed to be accounted for in the drawing, and they seem to form parts of definite structures with a resulting sharp and hard appearance. The same effect has been obtained time and again by others who have drawn details from the chromosphere to the furthest observed limits of the corona, but it is much more impressive to an observer who obtains the result himself.

The writer concludes that the corona is made up of numerous distinct structures, the interlacing and overlapping of which give any impression that exists of a globular or other general form. Examples of what is meant will be found in the following descriptions.

Certain structures appearing many times in the corona are described as follows :—

Petals : Great structures, almost triangular, having bases on the chromosphere and tapering to a point. Example in SW quadrant, P.A. 234°.

Fans : Diverging beams which take a variety of shapes and widths. Examples, SE quadrant, P.A. 162°, SW quadrant, P.A. 262°.

Rays : Very thin streamers. The best example in SE quadrant, P.A. 115°, was an exceedingly prominent and bright detail of the corona.

Arches : Generally well recognised features of coronæ. Example, surrounding the prominence near P.A. 52°.

Domes : Best described as filled-up arches. An example is seen under the arch mentioned above.

Loops : Very low arches. Only two seen, near fan quoted above in P.A. 262°.

The rays do not appear in latitudes higher than the normal sunspot zones.

Prominent features seen in short and medium exposure negatives are in most cases produced by superimposition of two or more fainter features. The bright petal with its apex at 165° is produced by the large petal in 176° and the fan at 160°. It may thus be referred to as a "spurious coronal feature." Another small prominent petal at 98° appears to be produced by the petal at 97° and the very broad fan whose extreme edges end at 118° and 141°. The spurious petal so produced is extremely prominent on all short exposures.

Thin dark lanes extending right to the chromosphere appear at 30° and 260°, the latter curving towards 250°. Such features have been a puzzle at many eclipses, suggestions ranging from obscuration by dark matter to actual absence of coronal material. In view of the remarks on coronal structures above, the writer wishes to draw attention to the possibility of the latter suggestion.

The Eastern Limb.—Even a rough examination of the negatives indicates a region of great complexity from 30° to 140°. There is a large branched prominence at 54°, under the eastern arm of which is a low prominence. Around the two prominences is a dome, and the whole surmounted by an arch. Then there is the bright spurious petal and the ray at 115°. Twisted rays above the spurious petal give a screw effect. At 111° is a thin straight prominence from which are two coronal arches drifting towards the north. The spurious petal at 97° has a cumulus cloud effect on the negative and raises the question of possible turbulence.

Solar Activity and the Corona.—The Zürich character figures give the solar maximum as occurring at 1937·4. The eclipse therefore occurred practically at solar maximum.

A longitudinal zone in which sunspot activity was greatest during 1937 *

* *Journ. R.A.S. Canada*, **23**, 183, 1938 April.

was on the eastern limb at the time of the eclipse. The zone between longitudes 170° and 210° was particularly active during solar rotations 1118, 1119 and 1120, being represented in the latter by four main centres of activity which were returns and the seats of many solar eruptions. The British Astronomical Association reported * that the group in lat. −16° long. 195° provided fine prominence displays when on both limbs, and that for the spot near lat. +10° long. 175° on June 11 there was "a very fine spectacular display consisting of a series of tree-like prominences which arose from the chromosphere as enormous clouds disappearing in succession." The twisted rays above the spurious petal on the eastern limb appear to originate in a latitude not much different to that of the spot in lat. +10° long. 205°.

The sunspots have been plotted on the drawing, as well as those on the invisible hemisphere which are just about to make an appearance on the eastern limb and those which have just disappeared round the west. Spots of the latter class can be seen close to the eastern limb and near the west limb in the SW quadrant. Allowance has been made for the original orientation error, so that sunspots, prominences and coronal features are correctly placed relatively.

Further interest is attached to the disturbed eastern corona and the sunspot zone by the observation of Newton † that in 1937 there had been a sequence of geomagnetic disturbances corresponding at the time of their respective commencements to solar longitudes between 80° and 180°, and by the writer ‡ that frequency maxima of the aurora australis for the same period occurred at solar longitudes 140° and 220°. The same solar region was facing the Earth during the great magnetic disturbances, varying cosmic ray intensities, radio fade-outs and brilliant auroral displays occurring between 1937 April 24 and 28. Most of the great terrestrial disturbances in 1937 and 1938 occurred when the same solar region faced the Earth.

Photometry of Corona.—An attempt was made to study the law of variation of brightness within the corona, but it was unsuccessful owing to difficulties with apparatus. No photometer being available, one was improvised, using a Weston photronic cell. At first this gave great promise, but later the cell showed a decadent effect which has not yet been explained. The work was therefore abandoned and only those results which could be considered unaffected by the cell's vagaries are plotted in fig. 1 showing isophotal lines for the eastern limb. As far as the work went it appeared that the coronal intensity varied as r^{-4}.

Suggestions for Future Photometry.—Evidence now accumulating suggests variations of brightness within the corona as a transient quantity, the value for different position angles probably varying from day to day. There are thus probably insufficient data for integrated values at present for theoretical work on coronal physics. Results of considerable interest might be obtained if it were possible to make photometric measures on individual features of

* *Journ. B.A.A.*, 1937 July.
† *Observatory*, 1938 May (Solar Notes).
‡ *Observatory*, 1938 October (Correspondence).

the corona, and to this end it is suggested that some such method as that adopted by Michie for obtaining nearly correct exposures at all parts of the corona on a single plate should be developed as far as possible.

Summary

The position of the observation spot on Canton Island and the observed contact times of the total solar eclipse of 1937 June 8 are given.

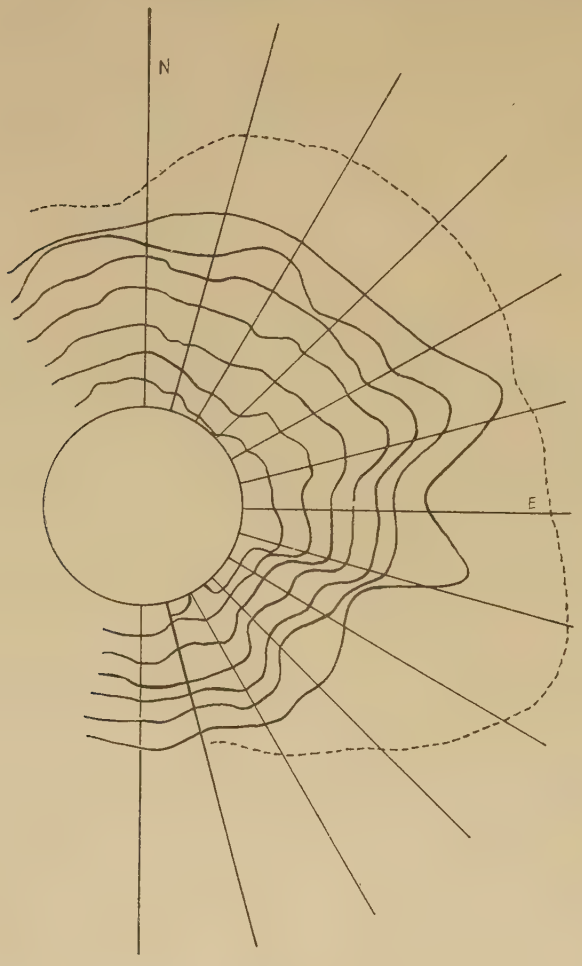


Fig. 1.

A discussion of the corona shows that it is made up of separate and distinct structures. The eastern corona appeared to be the most complicated, and attention is drawn to the fact that it probably had its origin in a very disturbed part of the Sun.

It is suggested that photometry of individual features of the corona might prove a fruitful source of research.

The writer wishes to express his thanks to Mr. C. B. Michie for the opportunity of accompanying the expedition and of later studying the photographs ; Professor F. J. M. Stratton for his interest and encouragement in the work ; and Dr. E. Marsden, Secretary of the Department of Scientific and Industrial Research, New Zealand, for considerable help in many directions.

THE PERSEUS CLUSTER

W. M. Smart, D.Sc., and A. Ali, M.A., B.Sc.

1. The parallelism of motion of sixteen B-type stars in Perseus was first pointed out independently by Eddington * and Boss † in the course of their investigations of the proper motions of the *Preliminary General Catalogue* which had just then appeared. The stars lie within the zone $+40°$ to $+50°$ in declination, and between $3^h\ 14^m$ and $5^h\ 18^m$ in right ascension; the values of the annual total proper motion, μ, are about $0''{\cdot}04$ in position-angles within a few degrees on either side of 140°. These stars are numbered 1 to 16 in Table I below, and they follow the order in Eddington's list. Later, Rasmuson ‡ added a further list of 29 stars between $0^h\ 29^m$ and $5^h\ 03^m$ in right ascension as possible candidates for membership of the cluster; these are numbered 17 to 45 in Table I (following the order in Rasmuson's list). The table contains three additional stars: Nos. 46 and 47 (2 Persei and 22 Aurigæ) are from Martin's list §; No. 48 was picked out from the new *General Catalogue* (*G.C.*) as possibly satisfying the required conditions.

For convenience we shall refer to the first sixteen stars in Table I as the Perseus stars and to the remainder as Rasmuson's stars.

2. Several determinations of the convergent point have been published during the last quarter of a century. A typical result is that of Rasmuson, namely,

$$A = 110° \pm 6°{\cdot}1\ ; \qquad D = -28°{\cdot}7 \pm 5°{\cdot}4,$$

in equatorial co-ordinates for 1900·0. In all such determinations the stars concerned are supposed to form a moving cluster. With this assumption Rasmuson's value for the cluster-motion, V, was found to be

$$V = 20{\cdot}0 \pm 6{\cdot}6 \text{ km./sec.}$$

Despite the extremely high value of the probable error of V no suspicions appeared to have been entertained as to the validity of the basic assumption of cluster-motion with which this group of stars was invested.

The present investigation follows the general lines of a similar paper (to be referred to as SI) by one of us.‖

3. The first step in the investigation is the determination of the convergent point, using the proper motions of the *General Catalogue*. These were corrected by means of Wilson and Raymond's first pair of formulæ.¶ The resulting position-angles, θ_0, of the proper motions are given in Table I together with the weights assigned according to the procedure described in SI.

* *M.N.*, **71**, 43, 1910.
† *A.J.*, No. 620, 1910.
‡ *Lund Medd.*, Ser. II, No. 26, 1921.
§ *M.N.*, **99**, 468, 1939.
‖ W. M. Smart, "The Scorpio-Centaurus Cluster", *M.N.*, **100**, 60, 1939.
¶ *Ap. J.*, **47**, 57, 1938.

TABLE I

No.	G.C.	Group	θ_0	Wt.	$\Delta\theta$	No.	G.C.	Group	θ_0	Wt.	$\Delta\theta$
			°		°				°		°
1	3923	1	131·5	2	−4·0	25	2241	7	110·8	1	− 8·2
2	3934	1	134·0	2	−1·0	26	2289	7	118·5	3	+ 2·8
3	3945	1	140·6	1	+5·6	27	2813	8	131·3	1	+ 5·9
4	4024	1	133·7	1	−2·1	28	2902	8	128·1	1	+ 1·9
5	4108	1	140·2	1	+3·5	29	3253	8	121·6	2	− 6·3
6	4122	2	134·0	1	−2·7	30	3390	8	123·7	2	− 5·3
7	4133	2	138·1	1	+1·3	31	3600	8	134·0	2	+ 2·5
8	4159	2	135·0	1	−2·2	32	3762	5	127·3	1	− 7·3
9	4287	2	137·1	2	−1·0	33	4041	1	137·3	3	+ 2·3
10	4427	2	141·9	3	+2·6	34	4052	5	126·9	1	− 9·8
11	4721	3	149·0	1	+7·4	35	4120	5	145·8	1	+ 8·5
12	4759	3	142·5	3	+0·5	36	4205	2	144·9	1	+ 7·4
13	4967	3	143·1	3	−0·6	37	4387	5	140·9	1	+ 1·5
14	5256	3	152·9	2	+6·8	38	4464	5	127·6	1	−12·2
15	6478	4	160·8	1	+4·3	39	4495	2	129·8	1	−10·1
16	6556	4	152·2	1	−4·9	40	4671	3	127·8	1	−11·9
						41	5216	...	140·2	2	− 3·5
17	618	6	105·6	2	+0·1	42	5687	...	135·0	2	−14·6
18	961	...	103·6	1	−6·9	43	5694	...	132·0	1	−16·3
19	1117	...	103·0	2	−4·1	44	5719	...	141·8	2	− 8·0
20	1120	6	101·7	1	−5·7	45	6226	4	158·7	3	+ 4·5
21	1426	6	106·7	1	−1·2						
22	1613	6	107·5	2	−1·0	46	2246	7	147·3	1	+26·7
23	2064	7	120·8	2	−2·9	47	6609	4	145·5	1	−10·7
24	2102	7	125·3	2	+6·0	48	4075	1	131·6	1	− 4·7

Considering the Perseus stars in the first instance, we see that their comparatively close concentration in the sky (within an area of about 200 square degrees) makes it impracticable to determine the convergent point of these stars by any of the usual methods. An empirical solution was first attempted, the results of which served to check the later values. The weighted mean value of θ_0, referred to the centre of position, C, of the stars concerned, defines a great circle on which the convergent point, P, should approximately be situated. If we can estimate the angular distance, λ, of C from P the calculation of the position of the convergent point can then be easily effected. For this purpose we used the radial velocities as a guide. Removing a K term of 5 km./sec. we found that the residual velocities appeared to have a mean value close to zero. This implied that the value of λ must be in the neighbourhood of 90°. Three positions of the convergent point (A, D) were then calculated with $\lambda = 75°$, 90° and 105° in succession. The results are as follows :—

	$\lambda = 75°$	$\lambda = 90°$	$\lambda = 105°$
	°	°	°
A	95·5	103·4	113·8
D	−18·3	−31·5	−44·1

The residuals found from the observed values of θ and the three sets of calculated values favoured the intermediate position.

To obtain a definitive position of the convergent point we utilise the stars of Table I, the earlier of Rasmuson's stars making it possible to obtain a solution of fair accuracy. The stars were arranged in eight regional groups the centres of which are given in Table II. The particular group to which an individual star is allocated is shown in the third column of Table I. Stars Nos. 18 and 19 were unaccountably omitted, and a group embracing Nos. 41–44 was subsequently discarded in the final solution.

The co-ordinates (A, D) of the convergent point are then found by means of the formulæ in SI (p. 66); the co-ordinates (a, d) of the poles of the great circles defined by θ_0, and the direction-cosines (l, m, n) of these poles, are given in Table II.

The results for epoch 1950·0—each group being accorded unit weight—are:

$$A = 108°{\cdot}0 \pm 2°{\cdot}8\,; \qquad D = -35°{\cdot}9 \pm 4{\cdot}1. \tag{1}$$

This position, which lies between the second and the third of the empirical solutions, is taken to be the convergent point.

The values of $\Delta\theta$, in the sense θ (observed) minus θ (calculated), are shown for the individual stars in Table I and for the groups in Table II.

TABLE II

Summary of the Groups

Group	α	δ	θ_0	$\Delta\theta$	a	d	l	m	n
	°	°	°	°	°	°			
1	49·6	47·6	135·4	−0·4	175·6	28·3	−0·877	+0·066	+0·474
2	53·3	47·9	138·2	0·0	176·9	26·5	− ·893	+ ·049	+ ·447
3	60·4	44·8	143·9	+0·7	177·6	24·7	− ·907	+ ·038	+ ·418
4	77·6	41·1	155·8	+0·3	184·1	18·0	− ·948	− ·066	+ ·309
5	51·6	33·9	133·7	−3·8	171·9	36·9	− ·792	+ ·112	+ ·600
6	13·9	60·2	105·8	−0·8	175·8	28·6	− ·875	+ ·063	+ ·478
7	26·4	51·8	122·9	+3·3	166·9	31·3	− ·833	+ ·193	+ ·519
8	40·2	53·2	125·6	−2·9	178·6	29·1	− ·873	+ ·024	+ ·487

When we remember that the annual total proper motion is only of the order of 0″·04, the great majority of the values of $\Delta\theta$ for the individual stars must be accounted as satisfactorily small in magnitude, and, in accordance with the proper motion criterion, we accept all the stars in Table I provisionally as possible members of the cluster, if such a cluster exists.

4. We now determine the stream-velocity, V, from the formula

$$V \sin \lambda = \frac{4{\cdot}74\mu}{p}, \tag{2}$$

in which λ is the angular distance of a star from the convergent point given by (1), μ is the total proper motion and p the spectroscopic parallax, the latter taken from Schlesinger's *General Catalogue of Stellar Parallaxes* (1935); the values of μ, λ and p are given in Table III. As noted below, the values

TABLE III

Parallax and Absolute Magnitude

No.	μ (Unit, 0″·001)	λ	Parallax (Unit, 0″·001)		m	Type	M
			Spectr.	Cal.			
		°					
1	47	96·3	7	9	5·38	B5	+0·1
2	42	100·5	5	8	5·30	B3	−0·2
3	36	100·4	6	7	5·08	B3	−0·8
4	32	99·3	7	6	5·30	B3	−0·8
5	39	98·7	8	7	4·94	B5	−0·8
6	40	99·2	6	7	5·64	B5	0·0
7	39	98·8	8	7	4·67	B5	−1·0
8	44	97·6	8	8	6·04	B9	+0·6
9	38	97·1	8	7	4·26	B5*p*	−1·5
10	47	96·1	15	9	3·10	B5	−2·1
11	35	94·7	6	6	5·34	B5	−0·6
12	38	88·4	6	7	2·96	B1	−2·8
13	40	93·4	13	7	4·03	B3*p*	−1·6
14	46	91·1	6	8	4·89	B3	−0·5
15	24	97·1	4	4	6·23	B3	−0·5
16	41	98·2	6	8	5·12	B3	−0·5
17	45	124·4	11	10	4·88	B8	−0·1
18	34	120·8	8	7	5·03	B9	−0·7
19	27	122·8	...	6	2·25	B0*p*	−3·9
20	35	122·7	...	8	5·54	B9	0·0
21	42	122·2	9	9	5·46	B9	+0·3
22	60	121·3	9	13	6·32	A0	+1·9
23	61	104·9	9	12	5·45	B8	+0·8
24	29	112·8	...	6	4·19	B0*p*	−2·0
25	23	113·9	4	5	5·49	B3	−1·2
26	40	117·6	11	8	3·44	B3	−2·0
27	44	107·5	12	8	5·56	A0*p*	+0·2
28	29	107·0	11	6	4·86	K5	−1·4
29	46	107·8	7	9	5·66	B8	+0·4
30	25	107·5	8	5	3·93	K0	−2·7
31	40	104·0	11	8	5·42	B5	−0·2
32	26	86·8	6	5	5·60	B9	−1·0
33	35	99·7	25	6	1·90	F5	−4·0
34	50	87·8	12	9	5·64	A0	+0·4
35	71	87·4	11	13	5·60	A0	+1·2
36	33	97·5	6	6	5·52	B8	−0·6
37	41	88·5	6	7	5·57	B5	−0·1
38	61	87·2	14	11	5·57	A2	+0·8
39	31	94·1	6	6	5·64	B9	−0·6
40	51	96·5	7	9	5·92	K0	+0·8
41	55	70·9	8	11	5·32	A0*p*	+0·5
42	64	90·9	9	12	5·70	A0	+1·1
43	54	73·9	14	10	5·68	A0	+0·7
44	66	86·5	10	12	5·25	A0	+0·7
45	77	82·6	13	14	3·28	B3	−1·0
46	33	111·7	8	7	5·64	B9	−0·3
47	39	69·9	6	8	6·39	B9	+0·8
48	36	99·0	5	7	5·91	B5	0·0

of the spectroscopic parallaxes assigned by Stebbins and tabulated by Martin* were also employed. Using Schlesinger's values of p and considering all the stars in Table I we obtain, as the solution of (2),

$$V = 25{\cdot}8 \pm 0{\cdot}9. \qquad (3)$$

Considering only the stars of type B the solutions are, with Schlesinger's and Stebbin's parallaxes, $27{\cdot}1 \pm 1{\cdot}1$ and $26{\cdot}0 \pm 1{\cdot}1$ respectively. We take as the definitive value of V for the B-type stars

$$V = 26{\cdot}5 \pm 1{\cdot}1. \qquad (4)$$

With the value of V in (3) and the individual values of λ and μ we calculate the parallax, p, of each star; the results are shown in the fifth column of Table III.

A scrutiny of the spectroscopic and calculated parallaxes shows general agreement except, perhaps, in a few instances. Ignoring these we cannot avoid the conclusion that, so far as the parallax criterion is concerned, the evidence is not condemnatory to the assumption that all the stars concerned —or, at anyrate, the great majority—form a moving cluster.

A further remark may be made. The values of V in (3) and (4) are obtained by giving each star unit weight in the solution of (2), whatever the value of p may be, and it is easily seen that the values of V are somewhat inflated by the stars of small parallax. For example, if we consider the Perseus stars alone, the value of V is found to be 25·2 km./sec., each star being given unit weight, and 23·6 km./sec. when stars with parallaxes exceeding 0″·006 are given double weight. It would thus appear that the value of the "stream-velocity" in (3) and in (4) would be appreciably diminished by a system of weighting of the kind just mentioned.

5. The last three columns of Table III give the apparent magnitudes, spectral types and absolute magnitudes, M, the latter obtained by means of the calculated parallaxes in the fifth column.

Table IV gives the mean absolute magnitude, $\bar{M}$, for each spectral subdivision and the corresponding value of the dispersion, σ.

TABLE IV

Mean Absolute Magnitudes

	B0, 1	B3	B5	B8, 9	A0
No.	3	10	10	12	8
$\bar{M}$	−2·9	−0·9	−0·6	0·0	+0·8
σ	0·9	0·5	0·8	0·6	0·5

The values of $\bar{M}$ in this table are in good accordance with our knowledge of the absolute magnitudes of stars of the spectral types concerned, and we conclude that the application of this criterion does not provide any evidence for distrusting the assumption of cluster-motion.

* *M.N.*, 99, 468, 1939.

6. As in SI the radial velocity criterion furnishes the crucial test as to the real kinematical character of the stars of the Perseus Cluster.

In Table V we deal with the stars according to their spectral subdivisions. The second column gives the H.D. numbers and the next two columns give the radial velocities and their probable errors taken from Moore's *Catalogue of Radial Velocities* * (*V* signifies that the radial velocity is variable and *V*? that it is suspected of being variable). The weights in the next column are assigned according to the following simple scheme :—

P.E.	0–1·0	1·1–2·5	2·6–5·0	> 5·0
Weight	3	2	1	0

If the stars form a moving cluster each star of a given spectral group should satisfy, within the limits of observational error, the equation

$$V \cos \lambda + K = \rho, \qquad (5)$$

where ρ is the observed radial velocity and K is the K term for the spectral group concerned.

We calculate K for each star ; on the assumption of cluster-motion all the values of K for a given spectral group should then, theoretically, be identical.

We have made the solutions for the following values of V :—

(i) $V = 26{\cdot}5$, obtained in (4) for the B-type stars, and $V = 25{\cdot}8$ for the stars of types A and later.

(ii) $V = 22{\cdot}0$ for all groups, a possible value suggested in part by the inferences of section 4.

The corresponding values of K are denoted in Table V by K_1 and K_2.

Considering first the section (*c*) of the Table, we see that the values of K_1 and K_2, which should theoretically be zero for these stars, are very far from satisfying this condition. There can be little doubt that these stars, with the possible exceptions of Nos. 28, 33 and 34, cannot be regarded as members of a moving cluster.

Consider now the Bo–B5 stars. If $\bar{K}_1$ and $\bar{K}_2$ denote the weighted mean value of K_1 and K_2, we obtain

$$\bar{K}_1 = 5{\cdot}2 \pm 0{\cdot}8 ; \qquad \bar{K}_2 = 4{\cdot}5 \pm 0{\cdot}8.$$

If we give each star unit weight, omitting the two stars to which zero weights are assigned, the results are :

$$\bar{K}_1 = 4{\cdot}6 \pm 0{\cdot}9 ; \qquad \bar{K}_2 = 3{\cdot}7 \pm 0{\cdot}9.$$

In estimating the probable errors just quoted the effects of inaccuracies in V and in the position of the convergent point given in (1) are not taken into account.

The residuals ϵ_1 and ϵ_2 defined for each star by

$$\epsilon_1 = K - \bar{K}_1 ; \qquad \epsilon_2 = K - \bar{K}_2$$

* *Lick Publ.*, **18**, 1932.

are given in the last two columns of the sections (*a*) and (*b*) of Table V. For cluster-motion these residuals should all be zero theoretically. The mean numerical values of ϵ_1 and ϵ_2 are 4·3 and 4·5 km./sec. respectively. Each of these quantities, which we denote simply by $\bar{\epsilon}$, represents the mean random radial speed of the stars concerned corresponding to the two values of V.

A glance down the K_1, K_2 or the ϵ_1, ϵ_2 columns in Table V (*a*) shows that the conditions for cluster-motion are far from being satisfied by all the stars of this spectral group. Nor is any improvement shown if we exclude the stars (Nos. 19 to 31) lying outside the region defined by the Perseus stars, for, considering $\bar{K}_1$, its value is reduced only from 5·2 to 5·0, so that the residuals ϵ_1 (and, in the same way, the residuals ϵ_2) remain substantially unchanged.

It is to be remembered that in these calculations no allowance has been made for any inaccuracy in the position of the convergent point. As the values of λ for the great majority of the stars are in the neighbourhood of 90°, the value of $\bar{K}$ deduced from the solution of (5) is sensitive to changes in the convergent point. With the comparatively small number of stars at our disposal we cannot pretend that, even if most of the stars do actually form a moving cluster, our convergent point has been determined with anything like the precision attainable in similar investigations. For this reason we do not attach much importance to the actual values of $\bar{K}_1$ and $\bar{K}_2$ derived above for the Bo–B5 stars. But the point that concerns us with reference to the kinematical character of the Perseus Cluster is that, with any reasonable changes in the convergent point, the values of $\bar{\epsilon}$ remain substantially the same. In other words, the ϵ-residuals show such a scatter as is inconsistent with the basic assumption of cluster-motion for all the stars concerned. For example, taking the stars in Table V (*a*) and using the intermediate of the three empirically determined convergent points (section 3) we find, for the weighted solution with $V = 26{\cdot}5$, that $\bar{K}_1 = 2{\cdot}5$ and $\bar{\epsilon}_1 = 4{\cdot}6$, the individual values of ϵ_1 being very much the same as those shown in Table V (*a*). The third of the three empirical positions gives much the same result as regards $\bar{\epsilon}_1$.

As regards the Bo–B5 stars our conclusion is that solutions for the K term are, to some extent, subject to uncertainty, but that the ϵ-residuals are of such a nature that the stars concerned do not, as a whole, show the characteristics expected of a moving cluster. This latter conclusion is unaltered if we insert in equation (5) a rotational term with the usual values of the rotational constants. The only substantial effect is to increase the value of K by 2·2 km./sec. If K has the normal value of about 4·5 km./sec., it would appear that the intermediate position of the convergent point in section (3) satisfies the conditions better than the definitive position used in the general calculations.

Consider now the group of B8 andB9 stars in Table V (*b*). The weighted solutions are :

$$\bar{K}_1 = 5{\cdot}5 \pm 1{\cdot}0\,; \qquad \bar{K}_2 = 4{\cdot}2 \pm 1{\cdot}0,$$

and the unweighted solutions are :

$$\bar{K}_1 = 6{\cdot}9 \pm 1{\cdot}0\,; \qquad \bar{K}_2 = 5{\cdot}5 \pm 1{\cdot}0.$$

TABLE V

Radial Velocity

(*a*) Spectral Types Bo–B5

No.	H.D.	Radial Velocity Obs.	P.E.	Wt.	K_1	K_2	ϵ_1	ϵ_2
1*	20315	− 0·2	± 2·4	2	+ 2·7	+ 2·2	− 2·5	− 2·3
2	20365	− 5·1	1·7	2	− 0·3	− 1·1	− 5·5	− 5·6
3*	20418	+ 3	4	1	+ 7·8	+ 7·0	+ 2·6	+ 2·5
4	20809	+ 4·8	1·3	2	+ 9·1	+ 8·4	+ 3·9	+ 3·9
5	21278	+ 7·1	2·1	2	+11·1	+10·4	+ 5·9	+ 5·9
6	21362	0	6*V*	0	+ 4·2	+ 3·5	− 1·0	− 1·0
7*	21428	− 0·8	3·0	1	+ 3·2	+ 2·5	− 2·0	− 2·0
9*	22192	+ 0·3	0·4	3	+ 3·6	+ 3·0	− 1·6	− 1·5
10	22928	−10	5*V*	1	− 7·2	− 7·7	−12·4	−12·2
11	24504	+ 9·8	0·6	3	+11·9	+11·5	+ 6·7	+ 7·0
12	24760	− 6	3	1	− 6·7	− 6·6	−11·9	−11·1
13*	25940	+ 4	3	1	+ 5·6	+ 5·3	+ 0·4	+ 0·8
14	27396	+ 1·3	0·8	3	+ 1·8	+ 1·7	− 3·4	− 3·8
15	34233	− 3	3	1	+ 0·3	− 0·3	− 4·9	− 4·8
16	34759	+ 5	5	1	+ 8·7	+ 8·1	+ 3·5	+ 3·6
19*	5394	− 6·8	0·6	3	+ 7·5	+ 5·1	+ 2·3	+ 0·6
24	10516	+ 1·4	1·3	2	+11·6	+ 9·9	+ 6·4	+ 5·4
25	11241	−11	6	0	− 0·2	− 2·0	− 5·4	− 6·5
26*	11415	− 8·1	0·5	3	+ 4·2	+ 2·1	− 1·0	− 2·4
31*	18537	− 4·1	2·1	2	+ 2·3	+ 1·2	− 2·9	− 3·3
37	22780	− 2·2	2·5	2	− 2·9	− 2·8	− 8·1	− 7·3
45*	32630	+ 7·8	1·3	2	+ 4·4	+ 5·0	− 0·8	+ 0·5
48	21071	+ 9·0	1·0	3	+13·1	+12·4	+ 7·9	+ 7·9

(*b*) Spectral Types B8 and B9

8	21551	+10	± 4*V*	1	+13·4	+12·8	+7·9	+8·6
17*	2772	−12	2·6*V*?	1	+ 3·0	+ 0·5	−2·5	−3·7
18	4636	+ 1·3	2·7*V*?	1	+14·9	+12·6	+9·4	+8·4
21*	6960	− 9·9	1·0	3	+ 4·2	+ 1·8	−1·3	−2·4
23*	10390	− 1·9	1·6	2	+ 4·9	+ 3·7	−0·6	−0·5
36*	21699	+ 1·2	1·0	3	+ 4·7	+ 4·1	−0·8	−0·1
39*	23300	+ 1·3	0·5	3	+ 3·1	+ 2·8	−2·4	−1·4

(*c*) Spectral Types Ao and later

No.	H.D.	Obs. R.V.	P.E.	Wt.	K_1	K_2	Type
22	8003	−18	± 3	1	− 4·6	− 6·6	Ao
27	14392	− 2·5	1·6	2	+ 5·2	+ 4·1	Ao*p*
34*	20995	+ 2·4	1·5	2	+ 1·4	+ 1·5	Ao
38	23193	+21·8	0·4	3	+20·5	+20·7	A2
41	27309	+12·2	0·8	3	+ 3·8	+ 5·0	Ao*p*
42	29526	+22·8	1·2	2	+23·2	+23·1	Ao
44	29722	+ 8·5	2·1*V*?	2	+ 6·9	+ 7·1	Ao
33	20902	− 2·4	0·1	3	+ 1·9	+ 1·3	F5
28*	14872	− 4·6	0·3	3	+ 2·9	+ 1·8	K5
30	17506	− 1·1	0·2	3	+ 6·6	+ 5·5	Ko
40	24240	+ 8·0	0·9	3	+10·9	+10·5	Ko

The two solutions reveal the effect of an arbitrary system of weighting applied to a small sample of stars. The values of $\bar{\epsilon}$ from the two weighted solutions are each 3·6 km./sec., which is reduced to 2·5 km./sec. if the values of ϵ are unweighted. The small value of $\bar{\epsilon}$ suggests that cluster-motion is a possibility for several of the stars in the group. If we consider only the well-determined radial velocities, we see that the last four stars in the list satisfy the criterion for cluster-motion almost exactly, the weighted values of $\bar{K}_1$ and $\bar{K}_2$ being 4·2 and 3·0 km./sec.

7. The first feature of the discussion in the previous section is the demonstration that, according to the radial velocity criterion, *all* of the Perseus stars or *all* of the stars in Table I cannot be regarded as members of a moving cluster.

The second feature is the small value of the mean random speed, $\bar{\epsilon}$, of the B stars investigated, our value being about a half of that usually obtained in general studies of such stars.

These two results are independent of any reasonable changes in the position of the convergent point and in the value of the "cluster-velocity."

Our value of $\bar{\epsilon}$ may be interpreted in two ways :

(i) Its comparative smallness may be due to the fact that a proportion of the stars with which we are dealing do actually form a moving cluster, much more attenuated numerically, however, than has been generally supposed hitherto.

(ii) Our small value of $\bar{\epsilon}$ may be the result of a purely fortuitous selection of stars with rather smaller random radial speeds than usual.

Considering (i) we indicate in Table V with an asterisk those stars which have small values of ϵ_1 (or of ϵ_2), and which satisfy the other criteria ; the selection is made with some reference to the probable errors of the radial velocities. We omit No. 33, which is one of the few stars ruled out by the parallax criterion. The number of stars selected in this way is 16, of which 9 are of spectral types Bo to B5, 5 of types B8, B9 and 2 of later types. Referring to the values of $\Delta\theta$ in Table I for these stars we remark that their observed position-angles are in accordance with the definitive convergent point used throughout in the calculations. The corresponding value of V is then found to be 23·7 km./sec., somewhat smaller than the value obtained for all the stars in Table I ; this new value is practically independent of any moderate change in the position of the convergent point. The unweighted values of $\bar{K}_1$ for the Bo–B5 stars and for the B8, B9 stars are 4·6 and 4·0 respectively.

Of the 16 stars, 9 are within the region defined by the Perseus stars, and only 5 belong to Eddington's list.

Considering the interpretation (ii) we remark that the "cluster-motion" is not very much in excess of the solar motion frequently found for B-type stars, and that, although the convergent point is about 17° away from the usual position of the solar antapex, this is the result of the particular way by which the stars were selected according to the position-angles of their

proper motions. If, for simplicity, we consider the Perseus stars only (they occupy a comparatively limited area of the sky) and assume that the system of B stars within this area form a single drift, our selection may be regarded as pertaining to those stars within a limited sector of the drift-curve adjacent to its major axis, and with total annual proper motions in the neighbourhood of 0″·04. If this interpretation is correct we must further suppose that our selection has resulted by chance in forming a group of stars with a mean random speed notably less than is found for the B stars in general.

Our conclusions from the investigation are (*a*) that a considerable proportion of the stars in the so-called Perseus Cluster cannot belong to a moving cluster, and (*b*) that a nucleus of stars remains which may possibly be regarded as a cluster-formation.

Summary

1. The convergent point of 45 stars, assumed to belong to the Perseus Cluster, is found to be (108°, −36°); the "cluster-velocity" is about 26 km./sec.

2. The parallax and absolute magnitude criteria are in accordance with cluster-motion as regards the bulk of the stars.

3. The radial velocity criterion rules out a considerable proportion of the stars as members of a possible cluster.

4. The mean random radial speed of all the stars with measured radial velocities is about $4\frac{1}{2}$ km./sec., considerably less than the value usually attributed to the B stars in general.

5. The paper is not definitely conclusive as to the real kinematical characteristics of the stars investigated; it is possible that a nucleus of stars, indicated in Table V, may form a moving cluster.

University Observatory,
Glasgow, W. 2:
1940 *February* 26.

THE CORONA BOREALIS CLUSTER

A. Ali, M.A., B.Sc.

(Communicated by the Director, University Observatory, Glasgow)

1. In *Lund Medd.*, Ser. II, No. 26, 1921, Rasmuson has investigated the communal motion of stars in the region of Corona Borealis. He remarks there that cluster motion in this part of the sky is probably doubtful. As only a few of the radial velocities and parallaxes of his stars were then known, he could not come to a definite conclusion. Since then, the publication of Moore's *General Catalogue of Radial Velocities* (1932) and Schlesinger's *General Catalogue of Stellar Parallaxes* (1935) has suggested a reinvestigation of this group of stars.

2. Rasmuson has given a list of 25 stars of large annual proper motions and mainly of later spectral type which are to be found in Table I, arranged according to R.A. as given in the *General Catalogue of Proper Motions* (1937).

TABLE I

No.	G.C.	θ_0	$\theta_0 - \theta_c$	a	d	l	m	n
		°	°	°	°			
1	19397	154·9	+ 16·1	305·9	25·1	0·531	−0·734	0·424
* 2	19414	171·2	+ 28·4	311·1	6·6	652	749	115
3	19550	144·9	+ 1·4	332·1	25·3	799	423	427
4	19666	137·5	− 4·6	345·8	21·4	902	228	365
5	19799	144·5	+ 1·4	341·1	17·8	901	308	306
* 6	19908	217·4	+ 92·2	330·1	32·7	730	420	540
7	20119	153·8	+ 6·8	334·3	16·5	864	416	284
* 8	20342	132·9	− 15·7	340·8	41·6	706	246	664
9	20532	150·5	+ 3·1	346·1	10·8	954	236	187
10	20591	144·4	− 0·8	320·6	35·6	628	516	582
11	20696	144·7	− 6·8	340·1	29·9	815	295	499
12	20914	128·4	− 13·4	310·7	50·5	415	482	772
* 13	20947	128·0	− 25·0	353·2	44·7	706	084	703
14	21044	145·8	− 8·4	350·6	22·6	911	150	384
15	21408	166·5	+ 11·8	332·2	13·0	862	454	225
16	21724	162·2	+ 7·5	333·8	17·7	855	421	304
* 17	22321	124·8	− 32·3	335·2	55·1	519	240	820
* 18	22636	189·9	+ 23·6	335·7	7·9	903	408	137
19	23446	172·9	+ 0·9	353·5	6·0	989	112	105
* 20	23837	152·6	− 21·1	359·7	26·7	893	− 004	449
* 21	23874	153·5	− 21·4	17·3	12·1	934	+ 290	210
* 22	23894	28·9	− 146·4	331·3	18·6	831	− 455	319
* 23	23962	211·3	− 35·6	334·3	24·4	821	395	413
24	24055	192·0	+ 16·1	350·8	11·2	0·968	− 157	194
25	24089	175·3	− 1·0	0·2	1·4	1·000	+0·004	024

The second column gives the *G.C.* numbers and the third column gives the observed position angle, θ_0, based on the values of $\mu_\alpha \cos\delta$ and μ_δ. These values were corrected by means of the first pair of the Wilson-Raymond formulæ.*

In the fifth and sixth columns, a and d are the equatorial co-ordinates of the poles of the great circles, defined by θ_0, in the northern hemisphere for each star. These were derived by means of the formulæ (2) and (3) in Smart's paper on "The Scorpio-Centaurus Cluster." † The last three columns give the values of the direction-cosines (l, m, n) of the poles (a, d).

3. If we assume the existence of cluster motion, the equatorial co-ordinates of the convergent point (A, D) can be obtained from the equation of condition (4) of the above-cited paper.

In the least-squares solution of this equation, all stars, except No. 6, were given unit weight, since the probable errors in θ_0 were usually of the order of 1°. No. 6 was omitted from the solution on account of the large probable error in θ_0.

After a preliminary solution, the co-ordinates of the convergent point, obtained for 1950·0, are

$$A = 270°\cdot3 \pm 3°\cdot3, \qquad D = -42°\cdot6 \pm 2°\cdot9.$$

Rasmuson's values are (he does not give the probable errors)

$$A = 276°\cdot0, \qquad D = -40°\cdot5.$$

4. With the above values of A and D, we can now calculate the position angle, θ_c, of each star, supposing its total proper motion to be directed towards this point. The fourth column of Table I gives the differences $\Delta\theta \equiv \theta_0 - \theta_c$. Keeping in view the large and accurately determined proper motions, the values of $\Delta\theta$ are so discordant as to suggest the absence of cluster motion. However, in order to apply the other criteria it is assumed, for a star to be a member of the cluster, that its total proper-motion circle should pass within 15° of the convergent point. (This limit of 15° is purely arbitrary and may be considered excessive; but, as will be seen later, it is immaterial to our results.) The corresponding values of $\Delta\theta$ are then given by $\sin\Delta\theta = \sin 15° \operatorname{cosec}\lambda$, where λ is the angular distance of the star from the adopted convergent point. As λ can be calculated for each star, these values of $\Delta\theta$ in conjunction with the residuals, $\theta_0 - \theta_c$, enable us to reject the stars which do not satisfy the condition imposed.

The application of the above test shows that the stars indicated by an asterisk in Table I should be ruled out at once. In the least-squares solution, all these, except the last three, were rejected beforehand as giving large residuals.

It may be mentioned here that stars Nos. 10 and 19 have been classified by Oort ‡ as high-velocity stars, and that Nos. 11 and 13 have been assigned by Smart, in his two papers §, to the Ursa Major Cluster, star No. 11 being

* *Ap. J.*, **47**, 57, 1938.
† *M.N.*, **100**, 60, 1939.
‡ *Groningen Pub.*, **40**, 1926.
§ *M.N.*, **99**, 441, 710, 1939.

regarded as doubtful. In our case, No. 13 was already rejected, and the rest were included in the general solution as satisfying the adopted position-angle criterion.

5. Out of the remaining 15 stars, two are without trigonometrical parallaxes. Thus there are 13 stars from which we can determine the space velocity V of the cluster from the relation

$$V \sin \lambda = \frac{4{\cdot}74\mu}{p}, \qquad (1)$$

where μ is the total proper motion and p the trigonometrical parallax. A least-squares solution of this equation gives us

$$V = 70{\cdot}7 \pm 8{\cdot}1 \text{ km./sec.}$$

Table II gives the relevant data which have been used in the solution of (1). The fourth column gives the weights which are roughly proportional to the inverse squares of the probable errors in p. The calculated values of the parallax p_c, on the basis of $V = 70{\cdot}7$ km./sec., are given in the penultimate column, while the last column gives the residuals $\Delta p = p - p_c$. The residuals in most cases are small enough and within the limits of the probable errors of the trigonometrical parallaxes. At least, all that one can say is that the parallax-test is not insuperably against the assumption of cluster motion except for a few of the stars.

TABLE II

No.	μ (0″·001)	p (0″·001)	Weight	λ	p_c (0″·001)	Δp (0″·001)
				°		
1	531	50 ± 7	2	66·3	39	+11
3	271	23 10	1	97·3	18	+ 5
7	254	27 7	2	101·0	17	+10
9	449	47 5	4	114·6	33	+14
10	639	38 6	3	58·0	50	−12
11	237	68 8	2	81·8	16	+52
12	386	24 5	4	46·2	36	−12
14	158	10 11	1	95·1	11	− 1
15	1326	78 7	2	65·3	98	−20
16	779	27 4	4	55·5	63	−36
19	1054	76 6	3	75·8	73	+ 3
24	659	55 11	1	64·2	49	+ 6
25	269	48 4	4	114·9	20	+28

6. The crucial test of radial velocities will now be applied, the relevant equation being

$$V \cos \lambda = \rho, \qquad (2)$$

where ρ is the observed radial velocity. In (2) the K term is omitted, as the stars concerned are of spectral types F to K. Table III gives the relevant details. The H.D. numbers are given in the second column, while the last column gives the residuals $\rho - V \cos \lambda$, where $V = 70{\cdot}7$ km./sec.

TABLE III

Radial Velocities

No.	H.D.	Sp. Type	ρ	P.E.	$V \cos \lambda$	Residual
1	126053	Go	−17·2	± 1·0	+28·4	−45·6
5	129580	Ko	− 9·0	0·8	−23·3	+14·3
7	132254	F5	−14·8	1·7	−13·5	− 1·3
9	136064	Go	−46·8	0·2	−29·4	−17·4
10	136202	Go	+53·5	0·2	+37·5	+16·0
11	137107	Go	− 6·5	...	+10·1	−16·6
12	138716	Ko	+47·7	0·2	+48·9	− 1·2
14	139798	Fo	− 2·3	1·8	− 6·3	+ 4·0
15	142860	F5	+ 6·5	0·5	+29·6	−23·1
16	145148	G5	− 3·3	0·7	+40·0	−43·3
19	157214	Go	−78·6	0·4	+17·3	−95·9
25	162003	F5	−10·2	0·4	−29·8	+19·6

If the stars form a cluster, the residuals should all be zero theoretically; whatever the observational errors may reasonably be, there can be no doubt that the extensive range in the residuals of Table III rules out any possibility that the stars in Rasmuson's list form a cluster.

Summary

In this paper, the motions of the stars in the environs of Corona Borealis as given by Rasmuson have been subjected to three different criteria for common cluster motion. It has been shown that the position-angle criterion is satisfied tolerably for about half of the stars by taking the convergent point as (270°·3, −42°·6). The parallax criterion is inconclusive as a whole, and the assumption of common cluster motion fails signally in the case of radial velocities. Hence it is concluded that no common motion exists in the case of the Corona Borealis stars.

University Observatory,
Glasgow, W. 2:
1940 *March* 9.

THE 61 CYGNI CLUSTER

P. C. Chaudhuri, M.Sc.

(Communicated by the Director, University Observatory, Glasgow)

1. The stars of the 61 Cygni Cluster with which this investigation deals are given in Rasmuson's * "Research on Moving Clusters." The group had previously attracted the notice of such observers as Boss †, Russell ‡, Plummer § and Corlin ||. In Rasmuson's work, information about 56 stars is available. We have utilised the accurate proper-motion data given in Boss's *General Catalogue* (1937), the *G.C.* numbers being shown in Table I.

Our method of investigating the convergent point of cluster motion is similar to that followed by Smart ¶ in the treatment of the Scorpio-Centaurus Cluster. The poles (a, d) of the great circles defined by the observed position angles, θ_0, of the total proper motions, μ, and the direction cosines (l, m, n) of these poles for individual stars, are shown in Table I.

TABLE I

No.	*G.C.*	θ_0	$\Delta\theta$	a	d	l	m	n	Sub-group
		°	°	°	°				
* 1	88	...	...	234·5	20·3	−·545	−·763	·346	...
2	92	94·1	−2·3	186·4	40·5	−·756	−·084	·649	I
† 3	588	94·4	9·0	182·4	13·2	−·973	−·041	·228	II
4	741	90·5	0·3	189·9	65·0	−·417	−·073	·906	I
5	1715	99·8	1·5	189·3	29·5	−·859	−·141	·492	III
6	2050	100·6	3·7	189·1	46·6	−·678	−·108	·726	III
* 7	2030	...	...	199·5	33·4	−·786	−·279	·551	...
* 8	2733	...	...	211·3	56·0	−·477	−·291	·829	...
9	3562	98·5	1·8	200·7	67·9	−·352	−·133	·927	III
* 10	4849	...	...	186·7	40·4	−·757	−·089	·648	...
11	4938	63·0	5·3	193·3	52·1	−·598	−·141	·788	II
12	6494	141·2	−4·7	196·2	28·6	−·843	−·245	·479	II
† 13	6966	23·0	8·3	195·9	5·3	−·958	−·263	·092	I
14	7161	15·3	2·7	190·4	2·5	−·983	−·181	·044	I
15	8960	358·8	4·9	11·3	0·8	·981	·195	·014	III
16	9280	352·9	0·8	8·4	2·7	·988	·146	·047	I
17	9523	252·8	5·7	11·2	4·8	·978	·192	·083	I
* 18	10040	...	...	19·3	13·1	·919	·322	·226	...
19	11121	219·6	−4·6	322·8	28·9	·696	−·529	·483	I
† 20	12449	328·0	7·1	16·4	15·9	·923	·271	·274	III

* *Lund Medd.*, Ser. II, Nr. 26, 1921.
† *A.J.*, No. 629, 1911 ; Nos. 633–34, 1912.
‡ *A.J.*, Nos. 635–36, 1913.
§ *L.O.B.*, 212, 1912.
|| *A.J.*, No. 782, 1920.
¶ *M.N.*, **100**, 60, 1939.

TABLE I.—*continued*

No.	G.C.	θ_0	$\Delta\theta$	a	d	l	m	n	Sub-group
		°	°	°	°				
21	13066	312·4	−3·7	4·3	6·9	·990	·074	·121	III
22	13627	290·0	−5·3	8·0	57·8	·528	·074	·846	II
23	14043	276·7	−2·8	16·3	80·1	·165	·048	·985	II
24	14464	295·6	−4·2	8·2	32·5	·836	·121	·537	II
* 25	15239	...	...	7·7	62·8	·453	·061	·889	...
26	16149	284·5	2·2	17·8	48·0	·637	·205	·743	II
† 27	17194	266·6	−6·8	4·2	41·6	·745	·055	·665	III
† 28	17487	269·3	−7·2	1·0	86·6	·059	·001	·998	III
29	17687	274·3	−3·9	354·0	77·9	·208	−·022	·978	III
30	17690	279·1	−0·8	4·8	25·7	·897	·076	·434	III
31	19949	292·7	−0·1	6·6	46·6	·682	·079	·726	II
32	19982	292·2	1·1	7·0	46·7	·681	·084	·728	I
* 33	20461	...	...	23·0	62·7	·421	·179	·889	...
† 34	20795	295·1	−6·8	7·4	52·2	·608	·080	·790	III
35	22464	309·5	−4·5	12·3	41·0	·738	·161	·656	II
36	24221	347·2	4·2	7·0	8·1	·983	·121	·140	II
* 37	24632	...	...	8·2	12·6	·966	·140	·218	...
† 38	24731	...	...	14·3	3·7	·967	·246	·065	I
* 39	26809	...	...	132·8	77·2	−·151	·146	·975	...
* 40	26816	...	...	190·4	73·7	−·276	−·051	·960	...
* 41	27289	141·4	19·8	193·1	36·9	−·779	−·182	·600	...
42	28959	46·1	2·5	191·0	36·7	−·788	−·153	·598	II
43	29066	44·1	3·0	188·2	30·1	−·856	−·121	·501	III
44	29509	52·4	0·0	187·2	38·4	−·778	−·098	·621	I
45	29640	...	...	90·0	39·2	·000	·775	·632	...
46	30314	86·8	5·8	59·3	86·4	·032	·054	·998	III
47	30817	123·1	−1·4	187·7	27·2	−·881	−·119	·457	II
† 48	31013	84·8	7·2	192·4	82·3	−·131	−·029	·991	III
† 49	31070	62·6	7·6	184·3	29·3	−·845	−·065	·489	II
* 50	31284	...	...	184·5	15·2	−·968	−·078	·262	...
51	31355	77·0	−2·6	228·7	76·2	−·660	−·751	·971	III
* 52	31899	...	...	188·3	79·4	−·990	−·144	·983	...
53	32003	75·0	0·3	201·1	64·8	−·933	−·360	·905	III
54	32342	94·1	3·6	192·1	79·7	−·978	−·210	·984	II
* 55	32557	...	...	185·7	56·8	−·995	−·099	·837	...
56	33120	78·2	−1·0	189·5	10·5	−·986	−·165	·182	III

2. A preliminary solution suggested that certain stars should be omitted on account of having $\Delta\theta = \theta_0 - \theta_c$, where θ_c denotes the position angle calculated with the adopted position of the convergent point, greater than 10°, or else of disagreement between the observed and computed radial velocities. Such stars are shown in Table I with an asterisk in column 1. The parallaxes and proper motions considered in conjunction with the observed radial velocities further indicate that the remaining stars should be classified into three different sub-groups and examined separately. The

classification of these stars appears in column 10, Table I. The equatorial co-ordinates (A, D) for the epoch 1950·0 of the convergent points for these sub-groups are then derived as follows :—

Sub-group	A	D
	°	°
I	97·7	−0·6
II	100·9	+3·2
III	97·2	+6·8

(1)

In the definitive solutions stars marked with an asterisk in Table III below are excluded.

3. Using the positions given in (1), we compute the position angles, θ_c, of the total proper motions, μ, and the angular distances, λ, for individual stars of the sub-groups. The values of $\Delta\theta$ are entered in column 4, Table I, while the observed position angles, θ_0, are shown in column 3. Angular distances, λ, appear in column 2 of Table III below. The probable errors in the proper-motion components are very small and consequently the probable errors in the observed position angles are also very small, being less than 1°·0 for all the stars concerned. If we put a modest limit to $\Delta\theta$ (*i.e.* if we reject stars having $\Delta\theta$ greater than 6°·0 numerically), we find that the stars marked with † in Table I should not be considered to be members of the different sub-groups on the basis of the position-angle criterion. It should be noted that the observed position angles have been computed from the proper-motion components given in *G.C.* after applying Wilson-Raymond's * corrections involving galactic rotation.

4. The cluster motion, V, for the different sub-groups can be obtained from the observed radial velocities, ρ, as well as from the available parallaxes, p, and proper motions, μ, by least squares solutions of the following equations of condition :—

$$V \cos \lambda = \rho \qquad (2)$$

and

$$V \sin \lambda = \frac{4{\cdot}74\mu}{p}. \qquad (3)$$

As our stars are of later types than B, no K term is required in (2).

The observed radial velocities, ρ (columns 3, 9, Table III), and parallaxes, p (columns 4, 5, Table II), for the stars concerned are taken respectively from Moore's *Radial Velocity Catalogue* (1932) and Schlesinger's *Catalogue of Parallaxes* (1935). The weights for use in the equations of condition (2) and (3) are entered in columns 5 and 11, Table III, and also in column 6, Table II. The values of the total proper motions, μ, are given in column 2, Table II.

The results of a least squares solution for the cluster motion, V, are as follows :—

Sub-group	From (2)	From (3)
I	104·1 ± 2·5 km./sec.	103·4 ± 2·0 km./sec.
II	77·2 ± 2·0 ,,	76·7 ± 2·1 ,,
III	38·7 ± 1·2 ,,	37·7 ± 1·6 ,,

(4)

* *A.J.*, No. 1084, 1938.

TABLE II

No.	μ (unit : 0″·001)	Parallaxes, p (unit : 0″·001)				m	Type	M
		Cal.	Trig.	Sp.	Weight			
				Sub-group I				
2	575	26	45	...	2	5·77	G0	2·87
4	1387	63	72	63	3	5·71	K0	4·71
13	315	15	15	...	0	5·06	K0	0·91
† 14	1103	51	42	...	0·8	5·65	G5	4·19
16 *	241	12	...	...	...	5·08	K2	0·45
17 *	206	12	...	...	...	5·11	K2	0·58
† 19	797	56	42	38	0	6·95	G0	5·71
32	225	11	...	13	...	5·50	K0	0·80
38	244	12	...	...	...	5·92	F5	1·25
44 *	5206	302	299	300	11	5·57	K5	7·97
				Sub-group II				
3	337	21	23	17	0·7	6·35	G5	2·93
† 11	227	18	40	...	...	5·57	A5	1·89
12	844	78	68	85	4	4·85	G0	4·30
† 22	193	15	26	17	...	5·00	K0	0·83
23 *	188	15	15	...	...	7·32	F8	3·12
24 *	467	30	31	...	1	5·08	G0	2·46
26 *	1588	99	...	97	...	5·04	G5	5·01
† 31	274	18	27	27	2	5·98	F0	2·27
† 35	602	52	110	100	...	3·00	G0	1·56
36	207	15	...	18	...	5·19	A2	1·14
42	463	38	39	42	3	2·64	K0	0·56
47	4698	313	288	280	3	4·74	K5	7·22
† 49	261	17	20	33	2	5·42	F8	1·54
† 54	556	38	16	11	2	8·30	G0	6·20
				Sub-group III				
5 *	299	37	33	68	2	2·80	A5	0·67
† 6	822	105	87	80	2	5·10	F8	5·21
9 *	237	36	31	36	3	5·85	F0	3·64
15 *	373	57	47	...	1·5	5·05	F2	3·82
20 *	328	42	51	...	3	5·17	F0	3·29
21 *	188	23	...	...	...	5·44	F2*p*	2·25
27	188	23	21	...	1	4·02	A2	0·84
28	256	31	39	25	1·5	6·15	F5	3·64
29	269	33	...	37	...	2·95	K0	0·55
30 *	183	22	...	29	...	6·02	F5	2·77
34	193	28	33	28	4	3·72	F0*p*	0·97
43 *	177	25	26	22	1	5·07	A5	2·07
46	232	39	34	20	0·8	6·80	F8	4·73
48	277	41	42	34	4	3·70	A2	1·75
† 51	204	29	40	39	2	5·85	G0	3·15
† 53	209	27	72	53	2	5·59	G0	2·75
† 56	234	29	93	83	0	6·55	K2	3·25

The agreement between the two sets of values for V is in accordance with the assumption that the stars under investigation are members of three different moving clusters.

5. We now calculate the parallax, p, and absolute magnitude, M, for each star by means of the usual formulæ. The values of p and M together with spectral types are given respectively in columns 3, 9 and 8 of Table II. For comparison, the values of the spectroscopic parallaxes are also entered in column 5. Hertzsprung-Russell diagrams can then be drawn on the basis of the information of columns 8 and 9.

The parallax criterion rules out stars Nos. 2, 19 (Sub-group I), Nos. 11, 35, 54 (Sub-group II), and Nos. 6, 53 and 56 (Sub-group III). Omitting them, we find that the sum of the residuals, p cal. $-p$ trig., is $0''{\cdot}023$ (4 stars, Sub-group I), $0''{\cdot}065$ (8 stars, Sub-group II) and $0''{\cdot}071$ (11 stars, Sub-group III), without regard to sign. The residuals are, therefore, in good agreement with the average probable error of a determination of a trigonometrical parallax.

If we draw Hertzsprung-Russell diagrams (not shown here), we find that the stars of our list fit in the giant and dwarf branches of that diagram, the majority being in the main sequence. In other words, neither the parallax criterion nor the spectral-absolute magnitude relation leads us to oppose the assumption of cluster motion for most of the stars in each of the three sub-groups.

6. We shall now consider the radial velocities. The residuals, $\Delta\rho \equiv (\rho_0 - \rho_c)$, have been shown in columns 6 and 12, Table III. The probable errors in the observed radial velocities are very small (Table III). Remembering this fact, and also bearing in mind that the stars are scattered over the sky, we assign a limit to $\Delta\rho$ in order to distinguish a cluster star. In other words, we shall reject those stars whose $\Delta\rho$ exceeds numerically 7 km./sec. In this way we are led to conclude that the stars marked with † in Table III should be definitely discarded as not being members of different sub-groups.

7. Combining all our observations concerning position angles, radial velocities, parallaxes and spectral-absolute magnitude relation, we find that for the stars of Sub-groups I and II the values of $\Delta\rho$ show such a considerable spread that the assumption of cluster motion cannot be entertained, though the criteria other than the radial-velocitiy criterion are not antagonistic to the assumption for these two sub-groups. Further, the observed radial velocities of stars Nos. 17 and 44 exceed Oort's minimum in his classification of the high-velocity stars * (62 km./sec.). The space velocities of the stars of these two sub-groups freed from the effect of solar motion (19·5 km./sec. towards $\alpha_0 = 270^\circ$, $\delta_0 = +30^\circ$) are respectively 87·7 km./sec. towards $A_0 = 99^\circ{\cdot}2$, $D_0 = +5^\circ{\cdot}6$, and 62·2 km./sec. towards $A_0' = 103^\circ{\cdot}9$, $D_0' = +13^\circ{\cdot}0$. Thus the positions of the convergence of the absolute motions, too, are in no way contrary to the conception that these stars are in a sense high-velocity stars satisfying Oort's additional criterion of preferential motion towards the apex of drift I. The average residuals $(\rho_0 - \rho_c)$, which are respectively 15·4 km. and 10·5 km. without regard to sign, support this view also.

* *Gron. Publ.*, 40, 1926.

TABLE III

No.	λ	ρ_0	P.E.	Wt.	$\Delta\rho$	No.	λ	ρ_0	P.E.	Wt.	$\Delta\rho$
						Sub-group I					
† 2	94°·7	0·9	±0·5	2	9·3	17	48°·9	63·6	±0·3	3	− 5·4
† 4	88·8	16·8	1·1	1	14·6	† 19	40·5	27·0	1·1	1	−52·2
*† 13	76·2	56·7	0·5*V*	2	30·8	† 32	117·0	−66·4	0·5	2	−18·7
14	80·1	11·6	0·6	2	− 6·5	38	107·7	...	...	...	...
16	67·6	38·8	0·6	2	− 1·2	44	128·3	−65·1	0·2	3	0·0
						Sub-group II					
*† 3	87·8	19·0	0·3	3	16·1	31	111·7	−34·9	1·1	1	− 6·3
† 11	49·4	60·7	1·6	1	9·9	† 35	134·3	−70·8	0·3	3	−16·8
† 12	41·9	65·7	0·3	3	8·2	† 36	124·7	−56·3	2·0	1	−12·4
22	54·0	50·5	0·6	2	5·1	† 42	132·3	−10·5	0·1	3	42·5
23	52·8	...	...	...	...	† 47	112·8	−40·4	0·3	3	− 9·2
24	73·5	19·6	0·7	2	− 2·2	49	107·1	−18·9	0·5	2	3·9
26	80·8	15·0	0·2	3	2·7	54	113·2	−32·0	1·2	1	− 1·6
						Sub-group III					
5	77·4	6·8	±2·1	1	− 1·6	30	87·3	...	...	...	...
*† 6	72·6	4·5	0·3	3	− 7·1	34	123·0	−21·3	...	1	− 0·2
9	53·4	27·9	0·5	2	4·1	43	120·3	−22·3	0·6	2	− 2·8
15	53·5	19·2	0·7	2	− 3·8	*† 46	132·6	−17·9	1·2	2	8·3
20	72·2	10·6	0·5	2	− 1·2	† 48	123·9	− 7·0	5·0	½	14·6
21	90·0	7·0	3·0	½	7·0	51	120·3	−17·9	0·8	2	1·6
27	96·1	5·0	4·0	½	9·1	*† 53	109·2	−31·2	1·2	1	−18·5
28	95·8	− 7·3	1·9	1	− 3·4	* 56	85·1	Var.	...	...	...
† 29	96·2	−14·0	0·1	3	− 9·8						

As regards the stars of Sub-group III, the position of the convergent point (97°·2, 6°·8) and space-motion (38·7 km./sec.) are in fairly close agreement with Smart's convergent (91°·8, 8°·1) and space-motion (42·5 km./sec.) for the moving cluster in Taurus.* This leads us to suggest that the stars in Sub-group III (or at least some of them) are members of the Taurus Cluster. If this is so, the cluster must extend over a much larger area of the sky than has been previously supposed. To test this suggestion we compute, using Smart's convergent and space-motion for the Taurus Cluster, the values of $\Delta\theta$, $\Delta\rho$, λ, parallax and absolute magnitude. We also calculate the distance, R, of each star from Smart's adopted centre of the cluster. These quantities are shown in Table IV. A scrutiny of our results shows that 8 stars may well be allocated to the Taurus Cluster, the remainder (indicated by asterisks) being ruled out from possible membership by one or more of the criteria considered in the earlier part of the paper; for example,

* *M.N.*, 99, 3, 1939.

No. 15 must be regarded as a field star, since the value of $\Delta\theta$ (Table IV) is exceedingly large.

TABLE IV

No.	$\Delta\theta$	$\Delta\rho$	λ	Parallax		Type	M	R (parsecs)
				Cal.	Trig.			
	°		°	″	″			
5	− 2·5	− 5·2	73·6	0·035	0·033	A5	0·51	6·5
9	0·9	− 0·5	48·0	·036	·031	F0	3·61	7·1
* 15	12·4	− 4·4	56·0	·050	·047	F2	3·54	15·3
* 20	12·0	− 0·2	75·3	·048	·051	F0	3·56	14·4
21	1·4	8·5	92·0	·021	...	F2*p*	2·05	12·6
* 27	− 3·8	12·8	100·6	·021	·021	A2	0·67	11·3
28	− 8·4	− 1·0	101·2	·029	·039	F5	3·47	1·1
29	− 6·4	− 5·8	101·1	·031	...	K0	0·38	2·8
* 34	−12·4	3·7	126·0	·027	·033	F0*p*	0·84	2·2
43	− 1·5	− 3·3	116·5	·022	·026	A5	1·78	10·2
46	6·8	7·9	127·1	·032	·034	F8	4·35	4·6
* 48	7·5	14·5	120·4	·036	·042	A2	1·47	7·6
51	− 2·0	1·7	116·1	·025	·040	G0	2·87	3·9

Summary

1. The stars of the so-called 61 Cygni Cluster have been divided into three sub-groups, and the convergent point and stream motion of each are calculated.

2. It is concluded that cluster motion is absent from the first two sub-groups, the members of which are mainly "high-velocity stars," and that probably several stars in the third sub-group are members of the Taurus Cluster.

In conclusion, I have pleasure in expressing my gratitude to Professor Smart for his kind and helpful suggestions during the preparation of the paper.

University Observatory,
Glasgow, W 2.:
1940 *March* 14.

Erratum

M.N., **100**, 176, last term in equation (10), *for* $(1-\delta)$ *read* $(1-\epsilon-\delta)$.